



9: Two Digit Addition and Subtraction

Linking Assessment to instruction

The following charts outline suggested activities for each the instructional levels for the *Two-Digit Addition and Subtraction* assessment. They refer you to particular activities from the *Developing Number Concepts* (DNC) series of books. The DNC series includes both teacher-directed and independent activities specifically designed to meet the varying needs of students. The suggested activities in each DNC book are coded for easy access. For example, “1:2-23” refers to Book 1, Chapter 2, Activity Number 23.

Please note that these charts provide a quick-overview of activities for instruction. For complete background information, please refer to these other important resources:

Math Time: The Learning Environment by Kathy Richardson

Developing Number Concepts by Kathy Richardson

For Professional Development Opportunities

Contact Math Perspectives Teacher Development Center: www.mathperspectives.com

Suggested Reading

For additional information that will support your instruction, refer to these sections from *Developing Number Concepts Book Three: Place Value, Multiplication, and Division*.

- What You Need to Know About Place Value (p. 2–4)
- Developing a Sense of Quantities to 100 and Beyond (p. 57–59)
- Addition and Subtraction of Two-Digit Numbers (p. 99–103)
- A Classroom Scene (p. 104–107)

Providing Appropriate Instruction

When helping students develop proficiency with two-digit addition and subtraction, it is important to recognize that competency develops over time and is the integration of several mathematical skills. To be proficient, the student must understand that numbers are composed of tens and ones; they must know parts of numbers to 10; and they must be able to reorganize quantities of tens and ones. In addition, they need to move away from using models and understand what is happening when they combine and separate numbers to solve problems without models.

- **Teacher-Directed Small Group Work**

Choose 3 or 4 activities and present them in a 10-15 minute small-group session to provide the children with a variety of experiences. Do the same tasks for several days. Occasionally replace one of the tasks with a new one. Adjust the size of the numbers according to the children's responses. Usually work with just one group a day so you have time to interact with the children while they work at the Independent Stations.

- **Independent Station Work**

Give children opportunities to choose from among several activities. These choices should be available for several weeks. During this time, the children will be developing proficiency and moving to larger numbers, as they are able. Interact with the children as they work, supporting them and challenging them as needed.

TWO-DIGIT ADDITION AND SUBTRACTION

This assessment helps you analyze precisely what students know and need to learn when working with two-digit addition and subtraction. The assessment provides information about how students solve the problems and whether or not they need a model to do so.

Goal:

To determine if the student can use the concept of tens and ones to add and subtract two-digit numbers by mentally breaking them apart and reorganizing them into tens and ones.

Addition:

The problems are presented at 3 different levels of abstraction:

Solving problems with a model ($28 + 16$, $26 + 27$)

Solving problem without a model ($36 + 25$)

Solving a symbolic problem ($28 + 14$)

Subtraction:

The problems are presented at 3 different levels of abstraction:

Solving problems with a model ($33 - 14$, $53 - 27$)

Solving problem without a model ($41 - 26$)

Solving a symbolic problem ($34 - 16$)

Present a variety of activities, allowing students to experience two-digit addition and subtraction in many ways over several months while providing additional experiences, if needed, with place value and number combinations. Let their responses dictate the amount of instructional time you spend before moving on.

Two-Digit Addition and Subtraction

Begin focusing primarily on addition.

Many of the same activities are used at all levels, but the focus changes according to the needs of the students. The following experiences focus on both addition and subtraction. Work primarily on addition first, but give students experiences with subtraction so they can begin to see how they are related. Once students are proficient with addition, focus primarily on subtraction, but continue to provide some addition practice as well.

1: Solving Problems with a Model

The assessment first determines whether students can solve problems when they have a model to refer to but are asked not to touch or move the model.

Needs Prerequisite (N)

(N) – Touches or moves model for both problems, counts all, or makes 2 errors

At this level, the children are unable to solve the problems using tens and ones.

*To get additional information about what the student understands about tens and ones and to determine what instruction the student needs in order to be ready to work with two-digit addition, use **Assessment 7: Ten Frames: Addition** and **Assessment 8: Grouping Tens**. If students are having difficulty breaking numbers to 10 apart to make tens, use **Assessment 6: Hiding Assessment** to find out what they know about parts of numbers.*

The following activities help students develop an understanding that numbers are composed of tens and ones. Students should work with these experiences until they can combine tens and ones and can add 10 and take 10 away without counting.

The Grouping Games

3:1-1	<i>Introducing the Plus-One and Minus-One Games</i>
3:1-2	<i>The Grouping Games with Groups of Other Sizes</i>
3:1-3	<i>Plus or Minus Any Number</i>
3:1-4	<i>Regrouping Beyond Two Places</i>

INDEPENDENT ACTIVITIES	
3:1-32	<i>Lots of Lines, Level 1</i>
3:1-33	<i>Paper Shapes, Level 1</i>
3:1-34	<i>Yarn, Level 1</i>
3: 1-35	<i>Yarn Shapes</i>
3:1-36	<i>Containers, Level 1</i>
3:1-37	<i>Cover It Up, Level 1</i>

(continued on next page)

3:1-38	<i>Measuring Things in the Room, Level 1</i>
3:1-39	<i>Measuring Myself, Level 1</i>
3:1-40	<i>Comparing Myself</i>
3:1-41	<i>Making Trails</i>
3:1-42	<i>Building Stacks</i>

Needs Instruction (I)

(I) – Counts to solve 1 problem, or touches/moves the model 1 time, or solves 1 problem as though it was written down and is incorrect for 1 problem.

Students need instruction if they show some beginning but tentative understanding of tens and ones. Focus on addition of two-digit numbers using models so the students can see what happens when the numbers are combined or broken apart and recombined.

TEACHER-DIRECTED ACTIVITIES	
2:3-30	<i>Working with Ten-Shapes</i>
3:1-45	<i>Addition and Subtraction of Two-Digit Numbers</i>
3:1-46	<i>Story Problems</i>

The students should also work to strengthen their understanding of Numbers as Tens and Ones through the following learning experiences. The students should work with these experiences until they can combine tens and ones and can add 10 and take 10 away without counting.

INDEPENDENT ACTIVITIES	
3:1-32	<i>Lots of Lines, Level 1</i>
3:1-33	<i>Paper Shapes, Level 1</i>
3:1-34	<i>Yarn, Level 1</i>
3: 1-35	<i>Yarn Shapes</i>
3:1-36	<i>Containers, Level 1</i>
3:1-37	<i>Cover It Up, Level 1</i>
3:1-38	<i>Measuring Things in the Room, Level 1</i>
3:1-39	<i>Measuring Myself, Level 1</i>
3:1-40	<i>Comparing Myself</i>
3:1-41	<i>Making Trails</i>
3:1-42	<i>Building Stacks</i>

Needs Practice (P-, P, P+)

Students need practice if they have to figure out the combinations needed to make tens and leftover ones and do not always easily keep in mind the number of tens they have formed or they ignore the models and solve the problems as though they were written down.

(P-) – Ignores the tens and ones represented by the model and visualizes the problem as though it were written down for 1 or 2 problems, or makes an error.

Children who ignore the tens and ones represented by the models and visualize the problem as though it were written down for 1 or 2 problems need more practice to see how tens and ones work. They also need more practice if they make an error when solving the problems using tens and ones.

(P) – Children are at this level if they can reorganize the numbers into tens and ones but count to figure out the parts for both problems, no errors.

(P+) – Children are at this level if they use tens and ones to solve the problems but need to count to figure out the parts for one of the problems.

To get more information if students are having difficulty breaking numbers to 10 apart to make tens, use **Assessment 6: Hiding Assessment** to find out what they know about parts of numbers and **Assessment 7: Ten Frames** to see if they can use parts of numbers to make one ten and leftovers.

Use the following activities to give the students practice breaking numbers apart and reorganizing them into tens and ones. Focus on helping them think about what number they need to make a ten and the number of leftovers they will have. Ask, “What do you think it will be?” and then have the students check if necessary. If they count to figure out, use the cubes and focus on adding parts of numbers rather than adding on the number by counting one by one.

TEACHER-DIRECTED ACTIVITIES	
2:3-30	<i>Working with Ten-Shapes</i>
3:1-45	<i>Addition and Subtraction of Two-Digit Numbers</i>
3:1-46	<i>Story Problems</i>

INDEPENDENT ACTIVITIES	
2:3-34	<i>Two Ten-Shapes: Addition Station</i>
3:1-48	<i>Partner Add-It</i>
3:1-50	<i>Roll and Add</i>
3:1-52	<i>Add 'Em Up: Lots of Lines</i>
3:1-53	<i>Add 'Em Up: Paper Shapes</i>
3:1-54	<i>Add 'Em Up: Measuring Things in the Room</i>
3:1-55	<i>Add 'Em Up: Yarn</i>
3:1-56	<i>Add 'Em Up: Yarn Shapes</i>
3:1-57	<i>Add 'Em Up: Containers</i>
3:1-58	<i>Add 'Em Up: Cover It Up</i>
3:1-59	<i>Solving Story Problems</i>

Ready to Apply (A)

Students are “Ready to Apply (A)” when they know the number combinations needed to make tens and leftovers and arrive at the answer without having to count to figure anything out and without errors. Move on to 2. Solving a Problem without a Model activities if the student is not yet “Ready to Apply” at that level.

2: Solving a Problem without a Model (36 + 25)

The students who are able to solve the first two problems in *Assessment 9: Two-Digit Addition & Subtraction* without touching or moving the model are asked to solve a problem when the model is hidden.

Needs Prerequisite (N)

(N) – Students need a prerequisite if they remove the paper to look at the models and then touch or move the model in order to see how to reorganize them into tens and ones.

*They need to continue to use the activities listed in the previous section **using models** until they can solve problems by looking at but not touching the model. Provide practice using the same activities as listed above with the emphasis now on moving away from dependence on the model.*

Needs Instruction (I)

(I) – Students are at this level if they remove the paper and look at the model in order to imagine what will happen to the numbers when they reorganize them into tens and ones.

*Work with the students with the activities listed above until they can easily solve the problems without touching the models. Then, begin asking them to try and solve parts of the problem **without looking at the models and then check** using the models. It will be helpful to some students if you **begin by adding single-digit numbers or work with tens only until it is easy for them to add these numbers without a model.***

Needs Practice (P-, P)

Students need practice if they can solve the problem without the model but are not yet proficient reorganizing the tens and ones to get to the answer.

(P-) – Students at this level ignore the tens and ones and try to solve the problem by visualizing it as though it were written on paper.

They should work with the models until they understand what is happening and can solve problems by mentally organizing numbers into tens and ones.

(P) – Students at this level can solve the problem but need to figure out the parts of the numbers in order to do so.

Use the same activities listed above. Focus on asking the children to think about what is needed to make a ten and the number of leftovers they will have. Have them solve parts of the problem without looking at the models. If necessary, have them check using the models.

TEACHER-DIRECTED ACTIVITIES	
2:3-30	<i>Working with Ten-Shapes</i>
3:1-45	<i>Addition and Subtraction of Two-Digit Numbers</i>
3:1-46	<i>Story Problems</i>

INDEPENDENT ACTIVITIES	
2:3-34	<i>Two Ten-Shapes: Addition Station</i>
3:1-48	<i>Partner Add-It</i>
3:1-50	<i>Roll and Add</i>
3:1-52	<i>Add 'Em: Up Lots of Lines</i>
3:1-53	<i>Add 'Em Up: Paper Shapes</i>
3:1-54	<i>Add 'Em Up: Measuring Things in the Room</i>
3:1-55	<i>Add 'Em Up: Yarn</i>
3:1-56	<i>Add 'Em Up: Yarn Shapes</i>
3:1-57	<i>Add 'Em Up: Containers</i>
3:1-58	<i>Add 'Em Up: Cover It Up</i>
3:1-59	<i>Solving Story Problems</i>

Ready to Apply (A)

(A) – Students have reached the desired level of proficiency when they solve problems without the need for a model, without having to count to figure anything out, and without errors.

Move on to 3. Solving Symbolic activities if the student is not yet “Ready to Apply” at that level.

TEACHER-DIRECTED ACTIVITIES	
2:3-30	<i>Working with Ten-Shapes</i>
3:1-45	<i>Addition and Subtraction of Two-Digit Numbers</i>
3:1-46	<i>Story Problems</i>

3: Solving Symbolic Problems

This part of the assessment focuses on the student’s ability to solve symbolic problems correctly and to demonstrate an understanding of the process used by showing the process with a model.

Needs Prerequisite (N) and Needs Instruction (I)

(N) – Gets the answer wrong and is unable to show the process they used.

(I) – Gets the correct answer but can’t show the process used.

Needs Practice (P)

The following activities are many of the same used previously, but emphasize interpreting symbolic problems with models.

(P-) – Is able to solve the problem and shows what they did with the model, but makes an error.

(P) – Is able to reorganize the numbers into tens and ones but counts to figure out the parts for both problems.

The goal is to work with the activities to develop proficiency with the process of using tens and ones to solve symbolic problems. Ask the students to describe how many are needed to make a ten and leftovers.

If students count to determine the number needed, have them use models and describe the parts they use to make the ten and what parts are leftover. When they can describe the parts with ease when they are using a model, ask them to imagine the parts they need by just looking at the written symbols. Continue to alternate between using the models and not using the models depending on how proficient the students are.

TEACHER-DIRECTED ACTIVITIES	
3:1-45	Addition and Subtraction of Two-Digit Numbers
3:1-47	Figure it Out

INDEPENDENT ACTIVITIES	
Have the children record, not only the answer, but how they got the answer as well.	
3:1-48	Partner Add-It
3:1-50	Roll and Add

Ready to Apply (A)

(A) – Is able to use tens and ones without counting or using a standard algorithm and shows with a model how the process used works.

If the student is “Ready to Apply” for addition problems, move on to an emphasis on subtraction. If the students are able to solve a variety of problems requiring two-digit addition or subtraction, they can begin working with forming and counting hundreds.

Additional Notes on Two-Digit Subtraction

Needs Prerequisite or Needs Instruction

It is not typical for a child to need a prerequisite for subtraction if they are doing well with addition using tens and ones. Help them make the connection to what they know about tens and ones.

Have those students who are struggling with subtraction and are not ready to work with subtraction independently, continue to work with the addition stations described in the previous section, Two-Digit Addition.

Needs Practice

The students should work with subtraction of two-digit numbers using models to see what happens when the numbers are combined or broken apart and recombined. Make sure the student understands they are taking away from the whole number and do not need to build the number they are subtracting. Some children will have used counting backwards to subtract and may be reluctant to give that method up. Begin by taking groups of ten away such as $34 - 10$ or $52 - 30$ to help them see there are more efficient and easier ways to solve problems than counting back. Taking groups of tens away will also help the students realize that it can be easier to take away from a number if they do not break it into tens and ones to subtract.

When asking children to solve a problem like $43 - 24$, the children will often break 43 into 40 and 3 to take the tens away and will describe what they did by saying, “I took 2 tens from 40, and now I have 2 tens left.” Focus them on the fact they still have some ones by asking, “How many did you start with? Tell me the whole number, not just the tens.” “So when you take 2 tens from 43, how many do you have left?”

Including the ones in the answer may not be easy for some children who are focused on taking tens from tens. They may need to be reminded for quite a while before they automatically include the ones in their answer.

Children take away the remaining ones in different ways (in this problem, they need to take away 4.) Some will break up a ten and then combine what is left with the 3 ones.

“I took the 4 from the ten and that left 6. Then I put the 6 with the 3 ones and that made 9. I have 1 ten and 9. That makes 19.”

Some will take away as many of the ones as they can and then take the rest from the tens.

“I can take these 3 ones away, but I still have to take away one more. I take it from the ten and now I have 19 left.”

Children generally have more difficulty solving subtraction problems than addition problems, so continue to have them use models to see what happens to the numbers until that is easy. Then ask them to say what they think will happen before they move the models and have them check and see. Eventually, they will be able to subtract easily by just looking at the models without touching or moving them. At this point, you can move on to covering the models. Let the children uncover the model if they have tried to solve the problem and realize they need to look. Alternate between showing the models, covering the models, and not using the models, depending on how proficient the students are.

The following activities focus on subtraction.

INDEPENDENT ACTIVITIES	
2:3-34	<i>Two Ten-Shapes: Subtraction</i>
2:3-35	<i>A Ten-Shape and More: Subtraction Station</i>
3:1-49	<i>Partner Take-Away</i>
3:1-51	<i>Roll and Subtract</i>
3:1-59	<i>Solving Story Problems</i>

When children are proficient subtracting by “taking away”, they can begin working with subtraction as the difference between two quantities. Use the following activities.

3: 1-32	<i>Lots of Lines, Level 2</i>
3: 1-33	<i>Paper Shapes, Level 2</i>
3: 1-34	<i>Yarn, Level 2</i>
3:1-36	<i>Containers, Level 2</i>