



1: Counting Objects Assessment

Linking Assessment to instruction

The following charts outline suggested activities for each instructional level of the *Counting Objects* assessment. They refer you to particular activities from the *Developing Number Concepts* (DNC) series of books. The DNC series includes both teacher-directed and independent activities specifically designed to meet the varied needs of students. The suggested activities are coded for easy access. For example, “1:2-23” refers to Book 1, Chapter 2, Activity Number 23.

Please note that these charts provide a quick-overview of activities for instruction. For complete background information, please refer to these other helpful resources:

Math Time: The Learning Environment by Kathy Richardson

Developing Number Concepts by Kathy Richardson

For Professional Development Opportunities

Contact Math Perspectives Teacher Development Center: www.mathperspectives.com

Suggested Reading

For additional information that will support your instruction, read the following sections from *Developing Number Concepts Book One: Counting, Comparing and Pattern*:

- What You Need to Know About Beginning Number Concepts (p. 2-4)
- Goals for Children’s Learning (p. 6-7)
- Meeting the Range of Needs (p. 8-11)
- Classroom Scenes (p. 12-21)
- About the Activities (p. 22-23)

Providing Appropriate Instruction

When helping children develop proficiency with counting objects, it is important to recognize that competency develops over time. Present a variety of activities, allowing children to experience counting in many ways over several weeks. This will help them make generalizations and integrate their ideas about counting. Let their responses dictate the amount of time you spend before moving on.

- **Teacher-Directed Small-Group Work**

Choose 3 or 4 activities and present them in a 10-15 minute small-group session to provide the children with a variety of experiences. Do the same tasks for several days. Occasionally, replace one of the tasks

with a new one. Adjust the size of the numbers according to the children’s responses. Usually work with just one group a day so you have time to interact with the children while they work at the Independent Stations.

- **Independent Station Work**

Give children opportunities to choose from among several activities. These choices should be available for several weeks. During this time, the children will be developing proficiency and moving to larger numbers, as they are able. Interact with the children as they work, supporting them and challenging them as needed.

COUNTING OBJECTS

Part 1, Task One: Counting a Pile (of 4, 7, 12, 21, 32)

Needs Instruction (I, I-)

(I) – Is unable to keep track of what they counted.

(I-) – Is unable to count one-to-one.

Focus on one-to-one counting. Help the children see the need to keep track by working with numbers that are small enough for them to be bothered when they arrive at different answers when they count and recount groups of objects. Discuss: What do we need to do so we know which ones we have counted and which ones we have not yet counted?

TEACHER-DIRECTED ACTIVITIES				
		To 7	To 12	To 21+
1: 1-1	Slide and Check	•	•	
1: 1-2	Count and Dump	•	•	
1: 1-3	Making Towers	•	•	
1: 1-4	Counting Stories, Level 1	•	•	•
1: 1-5	Creations	•	•	•
1: 1-7	Grab-Bag Counting	•	•	•
1: 1-8	Grow and Shrink, Level 1	•	•	•
1: 1-9	Hide It	•	•	
1: 1-10	Hunt for It, Levels 1 and 2	•	•	•
1: 1-11	Peek and Count, Level 1	•	•	
1: 1-12	Find a Match, Levels 1 and 2	•	•	•
1: 1-13	Tell Me Fast	•	•	
1: 1-14	Break It Up	•	•	•

INDEPENDENT ACTIVITIES				
		To 7	To 12	To 21+
1: 1-22	<i>Creations Station</i>	•	•	•
1: 1-23	<i>Cover the Dots, Level 1</i>	•	•	•
1: 1-24	<i>Counting with the Number Shapes</i>	•	•	•

Needs Practice (P)

(P) – Counts and recounts to find out how many, or loses track and is off by one or two, or keeps track with difficulty. (May need to line up the counters first to keep track.)

To develop more facility, provide ongoing practice with activities requiring the children to determine how many in a variety of situations.

TEACHER-DIRECTED ACTIVITIES					
		To 7	To 12	To 21	To 32 & Beyond
1:1-2	<i>Count and Dump</i>	•	•	•	•
1:1-3	<i>Making Towers</i>	•	•	•	•
1:1-6	<i>Finger Counting</i>	•	•		
1:1-7	<i>Grab Bag Counting</i>	•	•	•	•
1:1-8	<i>Grow and Shrink Level 2</i>	•	•	•	
1:1-9	<i>Hide It</i>	•	•	•	
1:1-10	<i>Hunt for It, Level 2</i>	•	•	•	
1:1-12	<i>Find a Match Level 2</i>	•	•	•	
1:1-14	<i>Break It Up</i>	•	•	•	•
1:1-15	<i>Tall and Short</i>	•	•	•	

INDEPENDENT ACTIVITIES					
		To 7	To 12	To 21	To 32 & Beyond
1:1-29	<i>Grab-Bag Counting Station</i>				
1: 1-30	<i>Shape Puzzles</i>	Use 3-6 Puzzles	Use 3-10 Puzzles	Use 10-20 Puzzles	Create Puzzles holding more than 20
1: 1-31	<i>Line Puzzles</i>	Use 3-6 Puzzles	Use 3-10 Puzzles	Use 10-20 Puzzles	Create Puzzles holding more than 20
1: 1-33	<i>Grab a Handful</i>	•	•	•	•
1: 1-34	<i>Hide-It Station</i>	•	•	•	
1: 1-36	<i>How Long Is It?</i>	•	•	•	•
1: 1-37	<i>How Many Does It Hold?</i>	•	•	•	•
1: 1-38	<i>Sorting Colors</i>	•	•		
1: 1-39	<i>Sorting Collections</i>	•	•	•	•
1: 1-40	<i>Sorting Shape Puzzles</i>	•	•	•	•
1: 1-41	<i>Sorting Line Puzzles</i>	•	•	•	•

Ready to Apply (A-, A)

(A-) – Keeps track with ease but leaves out a number in rote counting sequence, **or** forgets how many they counted.

Continue to provide opportunities to emphasize the counting sequence from 10 to 20:

- Counting the days of the month on a calendar*
- Lunch count*
- Days of school (represented by placing a dot each day on ten frames)*
- Library books to return*
- Children in line*

(A) – Knows the rote counting sequence, is able to keep track of the counters with ease, remembers what was counted, but does not line up the counters first. (We are determining the child’s ability to keep track of an unorganized group of objects that is not lined up.)

If children are ready to apply the skill of counting, provide a variety of opportunities for them to use what they know. The following are examples of ways you can involve children in counting.

- Working with Numbers to 12:*
- “How many books do we have about turtles?”*
 - “How many red buttons are in the button box?”*
 - “How many children are in your row?”*

- Working with Numbers to 21 or more:*
- “How many children brought their lunch today?”*
 - “How many paper clips long is your desk?”*

Part 1, Task Two: Making a Pile (of 5, 9, 18)

Needs Instruction (I)

(I) – Counts past the number asked for but doesn’t notice. (Is unable to hold the number in mind.)

Provide opportunities for children to count out a number of objects, beginning with numbers small enough for them to keep in mind as they count. Gradually increase the number, as the children are able.

TEACHER-DIRECTED ACTIVITIES				
		To 5	To 9	To 18
1: 1-4	Counting Stories	•	•	•
1: 1-5	Creations	•	•	•
1: 1-8	Grow and Shrink	•	•	Use two working space papers

INDEPENDENT ACTIVITIES				
		To 5	To 9	To 18
1: 1-21	Counting Boards	•	•	•
1: 1-22	Creations Station	•	•	•
1: 1-25	Roll-a-Tower Race	•	•	•

Needs Practice (P)

(P) – Counts out the requested number but makes an error, such as leaving out a number or losing track of an object.

Provide opportunities for children to count out a number of objects until they have developed meaning for the numbers and can hold them in their mind and count without making errors.

TEACHER-DIRECTED ACTIVITIES				
		To 5	To 9	To 18
1: 1-2	Count and Dump	•	•	Extend numbers as needed
1: 1-4	Counting Stories	•	•	Extend numbers as needed
1: 1-5	Creations	•	•	•
1: 1-8	Grow and Shrink	•	•	Use two working space papers

INDEPENDENT ACTIVITIES				
		To 5	To 9	To 18
1: 1-21	Counting Boards, Level 1	•	•	•
1: 1-25	Roll-a-Tower Race	•	•	•
1: 1-26	Make-a-Train Race	•	•	•
1: 1-28	Build a City	•	•	•
1: 1-32	Pick a Number	•	•	•

Ready to Apply (A-, A)

(A-) – Counts past the quantity asked for but self-corrects. The student realizes s/he has counted past the number, stops, and takes away any extra counters.

Continue to provide experiences counting out a number of objects as described in the previous charts.

(A) – Counts out the quantity asked for with ease and accuracy.

When children are ready to apply the skill of counting out a particular number, provide a variety of opportunities for them to use what they know. The following are examples of ways you can involve them.

“You need to get straws for everyone at your table. How many do you need to get?”

“Can you get a small whiteboard for everyone in the group? How many do you need?”

“We have 26 children in our class. Each child needs one piece of red paper. Would you count out 26 pieces of paper? “

Part 2, Tasks Three and Four: One More/One Less

(to/from 8, 12, 21, and over the decades to 100)

The Instructional Levels for the Part 2 Tasks, while determined separately, all use the same criteria.

Needs Prerequisite (N)

(N) – Counts to find out 3 (out of 3) times, or makes 3 errors.

Needs Instruction (I)

(I) – Knows without counting, or uses the rote sequence (counts under their breath) for 1 (out of 3).

Needs Practice (P)

(P) – Knows without counting 2 (out of 3) times, but counts to figure out one number, or makes one error.

Ready to Apply (A–, A)

(A–) – Knows without counting 2 (out of 3) times, but says the rote sequence (counts under his or her breath) 1 time, with no errors.

(A) – Knows one more without counting 3 (out of 3) times, with no errors.

The following experiences can be used for Task 3 and Task 4.

Provide ongoing experiences determining one more and one less, sometimes in sequence and sometimes not in sequence.

TEACHER-DIRECTED ACTIVITIES					
		To/From 8	To/From 12	To/From 21	To/From 100
<i>1: 1-3</i>	<i>Making Towers</i>	•	•	•	Provide opportunities for children to count large numbers using models like Ten Frames.
<i>1: 1-16</i>	<i>One More/One Less</i>	•	•	<i>EXT.</i>	
<i>1: 1-17</i>	<i>Give and Take</i>	•	•	•	
<i>1: 1-18</i>	<i>Hiding One More</i>	•	•	•	
<i>1: 1-19</i>	<i>Hiding One Less</i>	•	•	•	
<i>1: 1-20</i>	<i>Towers, Towers, Towers</i>	•	•	•	

INDEPENDENT ACTIVITIES				
		To 6	To 10	To 15
1: 1-27	<i>Build a Staircase</i>	•	•	
1: 1-35	<i>Give-and-Take Station</i>	•	•	•

Ready to Apply (A)

(A) – A child may be ready to apply for numbers to 8 but not for numbers to 12 and 21. Go on to the next level until the children are ready to apply for numbers to 32 and eventually to 100.



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Linking Assessment to instruction

2: Changing Numbers Assessment

Linking Assessment to instruction

The following charts outline suggested activities for each instructional level of the *Changing Numbers* assessments. They refer to particular activities from the *Developing Number Concepts* (DNC) series of books. The DNC series includes both teacher-directed and independent activities specifically designed to meet the varied needs of your students. The suggested activities are coded for easy access. For example, “1:2-23” refers to DNC Book 1, Chapter 2, Activity Number 23.

Please note – these charts provide a quick-overview of activities for instruction. For complete background information, please refer to these helpful resources:

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Suggested Reading

For additional information that will support your instruction, read the following sections from *Developing Number Concepts Book One: Counting, Comparing and Pattern*:

- What You Need to Know About Beginning Number Concepts (p. 2-4)
- Goals for Children’s Learning (p. 6-7)
- Meeting the Range of Needs (p. 8-11)
- Classroom Scenes (p. 12-21)

Providing Appropriate Instruction

When helping children develop proficiency with counting objects, it is important to recognize that competency develops over time. Present a variety of activities, allowing children to experience counting in many ways over several weeks. This will help them make generalizations and integrate their ideas about counting. Let their responses dictate the amount of time you spend before moving on.

Teacher-Directed Small-Group Work

Choose 3 or 4 activities and present them in a 10-15 minute small-group session to provide the children with a variety of experiences. Do the same tasks for several days. Occasionally replace one of the tasks with a new one. Adjust the size of the numbers according to the children's responses. Usually work with just one group a day so you have time to interact with the children while they work at the Independent Stations.

Independent Station Work

Give children opportunities to choose from among several activities. These choices should be available for several weeks. During this time, the children will be developing proficiency and moving to larger numbers, as they are able. Interact with the children as they work, supporting them and challenging them as needed.

CHANGING NUMBERS

Looking for relationships should be a part of children's work with counting. A focus on relationships will help children go beyond counting to developing meaning for the numbers they work with. When numbers are meaningful, children have a sense of their relative size. They know, for example, that when they are counting out 7 crackers, they are almost there when they have 5. They know that they have quite a few more to count if they want to end up with 10 crackers. In order to see the relationships between numbers, the child needs to see that one number is contained within another larger number.

In this assessment, two levels of understanding are assessed at the same time. One is the ability to change one number to another (Method for Changing Number) and the other is the ability to describe how many were added or taken away to make the new number (Describes Relationship).

Level 1: Method for Changing Number (to 6, to 10 and to 20)

The child is asked to change a number to another number 3 times, sometimes changing to a larger number and sometimes changing to a smaller number.

The purpose of Level 1 is to find out whether children understand that one number is contained in another number. There are two ways that children can demonstrate this level of thinking. One way is to add on or take off the appropriate number of objects. So, for example, a child would change 5 to 8 by getting 3 counters and adding them on, or she would change 8 to 6 by taking 2 counters away. Another way children can show an understanding of this concept is by counting on (or counting back) – but only if they are aware of the number added on or removed. To understand that one number is part of another number, the child must “see” both numbers when they are combined or separated. When a child counts on (or back) but is **not** aware of the number added on or taken away, from the child's point of view, the first number has merged into the other number and has “disappeared.” This reveals that the child is not yet fully cognizant of the parts that make up numbers.

Needs Prerequisite (N)

(N) – Unable to change the number 2 or 3 (out of 3) times—usually makes a new pile or adds the number asked for onto the original pile.

Children who make a new pile or add the number asked for onto the original pile do not see that one number is contained within another number. They think of each number as separate and unrelated to other numbers.

To get more information about what these students understand about counting, assess using *Assessing Math Concepts: Counting Objects*. This will help you determine whether they have the necessary foundation in counting.

Focus on counting activities where there is the potential for noticing relationships using numbers the children can count to easily.

TEACHER-DIRECTED ACTIVITIES	
1:1-8	<i>Grow and Shrink</i>
1:1-15	<i>Tall and Short</i>
1:1-10	<i>Hunt for It</i>
1:1-16	<i>One More/One Less</i>

INDEPENDENT ACTIVITIES	
1:1-40	<i>Sorting Shape Puzzles</i>
1:1-41	<i>Sorting Line Puzzles</i>

Needs Instruction (I)

(I) – Counts all 2 or 3 (out of 3) times. Changes the number correctly at least 2 (out of 3) times but needs to count all (starting with one) in order to do it.

At this stage, when asked to make a larger number, children understand that they need to add on to the pile, but they have to count the whole pile in order to do so. When asked to make a smaller number, they count to the number asked for and remove the extras, usually not paying attention to the particular number removed. Focus on asking them ahead of time whether they need to get some more or take some off to make another number to engage them in thinking about the relationships.

TEACHER-DIRECTED ACTIVITIES			
		To 6	To 10
1:1-8	<i>Grow and Shrink</i>	•	•
1:1-10	<i>Hunt for It</i>	•	•
1:1-15	<i>Tall and Short</i>	•	•
1:1-16	<i>One More/One Less</i>	•	•
1:3-8	<i>More-or-Less Counting Stories</i>	•	•
1:3-9	<i>Build a Stack</i>	•	•

INDEPENDENT ACTIVITIES			
		To 6	To 10
1:1-41	<i>Sorting Shape Puzzles</i>	•	•
1:1-41	<i>Sorting Line Puzzles</i>	•	•
1:3-22	<i>Counting Boards: Changing Numbers</i>	•	•

Needs Practice (P-, P, P+)

(P-) – Adds some, checks and fixes 2 or 3 (out of 3) times.

The children try to add on or take away without counting all the counters, but since they don't know how many to add on or take away, they "guess" an amount they think it might be and then check and fix it.

(P) – Counts on or back 2 or 3 (out of 3) times but does not know how many were added or taken away 2 or more times.

When children count on or back but are unaware of how many they added or took away, they need more practice – using smaller differences first and then moving to larger differences as they are able.

(P+) – Adds on a group; or counts on and says how many added 2 (out of 3) times.

Focus on identifying the number added or taken away when changing the number to another number.

TEACHER-DIRECTED ACTIVITIES	
1:1-8	<i>Grow and Shrink</i>
1:1-15	<i>Tall and Short</i>
1:1-16	<i>One More/One Less</i>
1:1-17	<i>Give and Take</i>
1:1-20	<i>Towers, Towers, Towers</i>

INDEPENDENT ACTIVITIES	
1:3-22	<i>Counting Boards: Changing Numbers</i>
1:1-35	<i>Give-and-Take Station</i>
1:1-38	<i>Sorting Colors</i>
1:1-39	<i>Sorting Collections</i>

Ready to Apply (A)

(A) – Adds on (or removes) a group with no errors; **or** counts on (or back) and says how many added (or removed) with no errors.

Level 2: Describing the Difference

When children first learn to change one number to another, they are focused on getting to the number and not on the quantity they needed to add or take away. Once the children are able to count on, count back, or count up, they should be asked to think about what they did to change the number. Most will need to actually change the number before they can tell you the difference.

Needs Prerequisite (N)

(N) – Unable to tell how many added or taken away, **or** makes 3 errors.

At this stage, the children's attention is on the number they are trying to make and not on the difference between the numbers. Continue with the activities described above until changing the number is easy enough for them and they begin to think about the difference between the two numbers.

Needs Instruction (I)

(I) – Is able to say the number added or taken away 1 time only, **or** makes 2 errors.

At this stage, the children are beginning to notice what they did to change the number and can say how many they added or took away for one of the numbers. Work with small numbers and small differences to help them become more aware of what they are doing to change the number. Ask them to consider ahead of time whether they need to get some more or take some off to make the new number. Continue with the activities described above at the instructional level identified for Method for Changing.

Needs Practice (P)

(P) – Says number added or taken away for all 3, but makes 1 error, **or** figures out the number added or taken away 2 or 3 (out of 3) times. May make an error.

At this stage, the children are able to figure out how many they added or took away 2 or 3 (out of 3) times. Focus on providing many experiences with small numbers and small differences until they know how many to add or take away without needing to figure it out.

TEACHER-DIRECTED ACTIVITIES	
1:1-8	Grow and Shrink
1:1-15	Tall and Short
1:1-16	One More/One Less
1:1-17	Give and Take
1:1-20	Towers, Towers, Towers

INDEPENDENT ACTIVITIES	
1:3-22	<i>Counting Boards: Changing Numbers</i>
2:1-8	<i>Grow and Shrink: Using the Plus (+) and Minus (-) Signs</i>
2:3-16	<i>Apartment Buildings</i>

Ready to Apply (A-, A)

(A-) – Says number added or taken away but checks 1 to 3 (out of 3) times.

At this stage, the children know the number they added or took away, but they lack confidence and need to check. Continue with the activities described above for “Needs Practice”. Help the children become aware when they know how many they added on or took away. Sometimes have them predict ahead of time what they think the number will be to help them see that they know some differences even before they count.

(A) – Says number added or taken away for 3 (out of 3) times, no errors.

Once children know the relationships between numbers within a certain range, give them experiences with larger numbers or greater differences.



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Linking Assessment to instruction

3: More/Less Trains Assessment

Linking Assessment to instruction

The following charts outline suggested activities for each instructional level of the *More/Less Trains* assessment. They refer you to particular activities from the *Developing Number Concepts* (DNC) series of books. The DNC series includes both teacher-directed and independent activities specifically designed to meet the varied needs of students. The suggested activities are coded for easy access. For example, “1:2-23” refers to Book 1, Chapter 2, Activity Number 23.

Please note – these charts provide a quick-overview of activities for instruction. For complete background information, please refer to these other helpful resources:

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Suggested Reading

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- What You Need to Know About the Concepts of More and Less (pp. 126-128)
- Goals for Children’s Learning (p. 132)
- Meeting the Range of Needs (pp.133-134)
- Classroom Scenes (pp. 134-142)

Providing Appropriate Instruction

When you are helping children develop proficiency comparing numbers, it is important to recognize that competency develops over time. Present a variety of activities, allowing children to experience the concepts in many ways over several weeks. This will help them make generalizations and integrate their ideas about number relationships. Let their responses dictate the amount of instructional time you provide before moving on.

- **Teacher-Directed Small-Group Work**

Choose 3 or 4 activities and present them in a 10-15 minute small-group session to provide the children with a variety of experiences. Do the same tasks for several days. Occasionally replace one of the tasks with a new one. Adjust the size of the numbers according to the children's responses. Usually work with just one group a day so you have time to interact with the children while they work at the Independent Stations.

- **Independent Station Work**

Give children opportunities to choose from among several activities. These choices should be available for several weeks. During this time, the children will be developing proficiency and moving to larger numbers, as they are able. Interact with the children as they work, supporting them and challenging them as needed.

MORE/LESS TRAINS

Several levels of understanding are assessed with the *More/Less Trains* assessment.

Level 1: Using One Train to Determine Another

The first part of the assessment uses Unifix cube trains that are lined up so the relationships between the trains are more visible. The children are assessed to see if they can use what they know about the number of cubes in one train to figure out the number of cubes in the second train. Seeing this relationship is a prerequisite for comparing the trains to determine how many more or less one is than the other.

Level 2: Finding the Difference between Trains

Once the student knows the number in both trains, s/he is asked to tell how many more (or less) one train is than the other.

Level 3: Finding the Difference between Groups

If the students are able to tell how many more or less one train is than the other, they will then be asked to compare groups that are not organized. When groups are not lined up, the relationships are less apparent, so comparing is more difficult.

Level 1: Using One Train to Determine Another

Needs Prerequisite (N)

(N) - Uses one train to figure out the other (0 out of 3) times.

At this stage, children do not use one train to help them figure out the other train. They count to find out how many every time, indicating that they see the numbers as separate entities.

*To get more information about their ability to see how one number relates to another, assess using **Assessment 2: Changing Numbers**.*

Needs Instruction (I)

(I) – Uses one train to figure out the other (1 out of 3 times), usually when the difference is 1.

At this stage, the children are just beginning to notice that they can use what they know about one train to help them figure out the other. They use the relationship between the trains just once. Most often they notice this when the difference is only one cube.

Needs Practice (P)

(P) – Uses one train to figure out the other (2 out of 3 times). May count all 1 time or make an error.

At this stage, the children use what they know about one train to help them figure out the other most of the time but still need more practice. Most often they will be able to use one train to figure out a longer train, but not a shorter train.

The following tasks can be used for children who need instruction as well as those who need practice. Focus on providing opportunities for children to look for parts of two groups that are the same. This will help them attend to what is different more easily.

TEACHER-DIRECTED ACTIVITIES	
1:1-20	<i>Towers, Towers, Towers</i>
1:3-1	<i>Is It More or Is It Less?</i>
1:3-2	<i>Stacks</i>
1:3-5	<i>Graph and See</i>

INDEPENDENT ACTIVITIES	
1:1-27	<i>Build a Staircase</i>
1:3-13	<i>Stack, Tell, Spin, and Win</i>
1:3-14	<i>Two-Color Grab-Bag Station</i>
1:3-20	<i>Sort and Compare Colors</i>

Ready to Apply (A)

(A) – Uses one train to figure out the other train (3 out of 3 times).

When children see that one number is related to another and can use that information, they are ready to begin learning particular relationships and how to determine how many more or less one number is than another.

Level 2: Finding the Difference between Trains

The process for learning to tell how many more and how many less is a long and complex one. Children may be able to tell how many more but not how many less. They may be able to tell how many more or less when the differences are one or two but not if larger. Be aware of these different competency levels as you provide experiences to children.

Needs Prerequisite (N)

(N) – Tells “how many more or less” 0 (out of 3) times (Usually tells the number in the group with more).

*To get more information about children’s ability to see relationships between numbers, assess with **Assessment 2: Changing Numbers**.*

Needs Instruction (I)

(I) – Tells “how many more or less” 1 (out of 3) times.

Needs Practice (P)

(P) – Tells how many more or less 2 times with 1 error, **or** matches the cubes to find the difference 2 or 3 times.

(P+) – Tells how many more or less 2 times and finds out by matching 1 time.

Notice that the instructional levels I, P, and P+ describe varying levels of proficiency comparing quantities. The activities for each instructional level are essentially the same but the questions, the size of the numbers and the differences between numbers can be varied to meet the children’s needs.

Children who are just beginning to notice the differences between numbers will need work with one more and one less and two more and two less before asking them to consider larger differences. When children find comparing trains difficult, use the language suggestions in the activities that children focus on what you want them to notice. Ask, “What do you have to do to make this train the same as that train?” or ask: “How many extras?”

Keep the size of the numbers and the differences small until they are successful. As they develop competency with each level, provide experiences with more challenging levels.

TEACHER-DIRECTED ACTIVITIES	
1:3-1	<i>Is it More or Is It Less? Level 2</i>
1:3-2	<i>Stacks, Level 2</i>
1:3-3	<i>Two-Color Grab Bag, Level 2</i>
1:3-5	<i>Graph and See, Level 2</i>
1:3-6	<i>Number Cards, Level 2</i>
1:3-7	<i>More-or-Less Spin It, Level 2</i>
1:3-9	<i>Build a Stack</i>
1:3-11	<i>More or Less</i>
1:3-12	<i>Roll and Spin</i>

INDEPENDENT ACTIVITIES	
1:3-13	<i>Stack, Tell, Spin, and Win</i>
1:3-15	<i>Comparing Lengths, Level 2</i>
1:3-14	<i>Two-Color Grab-Bag Station, Level 2</i>
1:3-18	<i>Comparing Handfuls, Level 2</i>
1:3-19	<i>Comparing Containers, Level 2</i>
1:3-10	<i>Grow, Shrink, and Compare (use 2 working space papers)</i>
1:3-20	<i>Sort and Compare Colors, Level 2</i>
1:3-21	<i>Comparing Numbers, Level 2</i>

Ready to Apply (A)

(A –) – Tells “how many more or less” (3 out of 3 times), with no errors.

Level 3: Differences between Groups

Needs Prerequisite (N)

(N) – Is unable to tell the difference between numbers (usually tells the number in the group with more).

Children who are able to tell the difference between the connecting trains that are lined up but cannot compare groups that are not lined up should work with groups that are just 1 or 2 more or less than the other. Ask them to think about how they could figure out the differences. This will give them the opportunity to consider matching the objects in the groups and seeing how many extras there are.

Needs Instruction (I)

(I) – Tells how many more or less 1 out of 3 times; may know the difference or match the cubes to find out.

Needs Practice (P, P+)

(P) – Tells how many more or less 2 times with 1 error, **or** they match the cubes to find the difference 2 or 3 times.

(P+) – Tells how many more or less 2 times and finds out by matching 1 time.

The same activities can be used for I, P, and P+. The size of the numbers and the size of the differences can be adapted to meet the children’s needs. Have the children experience some tasks that are not lined up along with those that are lined up.

TEACHER-DIRECTED ACTIVITIES	
1:3-4	<i>Spin and Peek, Level 2</i>
1:3-6	<i>Number Cards, Level 2 (vary by making groups instead of trains)</i>
1:3-8	<i>More-or-Less Counting Stories, Level 2</i>
1:3-10	<i>Grow, Shrink, and Compare (use 2 working space papers)</i>
1:3-11	<i>More or Less</i>
1:3-12	<i>Roll and Spin (vary by not connecting the cubes)</i>

INDEPENDENT ACTIVITIES	
1:3-15	<i>Comparing Lengths</i>
1:3-14	<i>Two-Color Grab-Bag Station, Level 2 (vary by not connecting the cubes together)</i>
1:3-16	<i>Comparing Shape Puzzles, Level 2</i>
1:3-17	<i>Comparing Line Puzzles, Level 2</i>
1:3-18	<i>Comparing Handfuls, Level 2</i>
1:3-19	<i>Comparing Containers, Level 2</i>
1:3-10	<i>Grow, Shrink, and Compare (use 2 working space papers)</i>
1:3-20	<i>Sort and Compare Colors, Level 2</i>
1:3-21	<i>Comparing Numbers, Level 2 (vary by not connecting the cubes)</i>

Ready to Apply (A)

(A –) – Tells how many more or less 3 (out of 3) times, with no errors.

Children who are ready to apply will be able to begin to learn relationships between numbers so they will be able to tell “How many more?” and “How many less?” one number is than another when the numbers are presented symbolically.



4: Number Arrangements Assessment

Linking Assessment to instruction

The following charts outline suggested activities for each instructional level of the *Number Arrangements* assessment. They refer you to particular activities from the *Developing Number Concepts* (DNC) series of books. The DNC series includes both teacher-directed and independent activities specifically designed to meet the varied needs of students. The suggested activities are coded for easy access. For example, “1:2-23” refers to Book 1, Chapter 2, Activity Number 23.

Please note – these charts provide a quick-overview of activities for instruction. For complete background information, please refer to these other helpful resources:

Math Time: The Learning Environment by Kathy Richardson

Developing Number Concepts by Kathy Richardson

For Professional Development Opportunities

Contact Math Perspectives Teacher Development Center: www.mathperspectives.com

Suggested Reading

For additional information that will support your instruction, read the following sections from *Developing Number Concepts Book Two: Addition and Subtraction*:

- What You Need to Know About Addition and Subtraction (p. xvii-xxii)
- Internalizing Number Combinations to 10 (p. 42-44)
- Goals for Children’s Learning (p. 44)
- Meeting the Range of Needs (p. 47)
- Classroom Scenes (p. 48-54)
- About the Activities (p. 54-55)

Providing Appropriate Instruction

When helping children develop proficiency with identifying and combining parts, it is important to recognize that competency develops over time. Present a variety of activities, allowing children to create and look for parts of numbers using many different models over several days or even weeks. This will help them make generalizations and integrate the idea that numbers are composed of parts into their thinking. Let their responses dictate the amount of time you spend before moving on to the next level.

- **Teacher-Directed Small-Group Work**

Choose 3 or 4 activities and present them in a 10-15 minute small-group session to provide the children with a variety of experiences. Do the same tasks for several days. Occasionally replace one of the tasks with a new one. Adjust the size of the numbers according to the children's responses. Usually work with just one group a day so you have time to interact with the children while they work at the Independent Stations.

- **Independent Station Work**

Give children opportunities to choose from among several activities. These choices should be available for several weeks. During this time, the children will be developing proficiency and moving to larger numbers, as they are able. Interact with the children as they work, supporting them and challenging them as needed.

NUMBER ARRANGEMENTS

This assessment determines where children are in the process of learning number combinations. Children go through 3 levels in learning to add numbers. They first learn to recognize small groups up to 5 without counting. Next, they begin to see these small groups within a larger number. Finally, they learn to recognize one part and count on from there or combine the parts to determine the total.

The assessment begins with the highest level (Combining Parts) to determine whether children can combine parts of numbers to find the total. If children do not combine parts but instead count all the dots, they are asked to identify whatever groups they can see within the larger number (Identifying Parts). If they don't see small groups within the number, they are shown small groups of up to 5 dots to see if they can recognize these groups when they are not a part of a larger number (Recognizing Small Parts).

Many of the same activities are used for all the levels, but the way children work with these activities will vary depending on their level of understanding and their interactions with the teacher while they are working.

Recognizes Small Groups

Needs Prerequisite (N)

(N) – Does not recognize any groups without counting, or makes 2 or more errors.

This indicates that children need to have opportunities to work with small groups of objects. Begin with numbers that are small enough for them to realize that they can recognize the groups without counting- for some children this will be a group of 2.

TEACHER-DIRECTED ACTIVITIES	
1:1-13	<i>Tell Me Fast</i>
1:1-10	<i>Hunt for It</i>
1:1-9	<i>Hide it</i>
1:1-12	<i>Find a Match</i>

Needs Instruction (I)

(I) – Recognizes 1 group without counting. May make 1 error.

Needs Practice (P)

(P) – Recognizes 2 or 3 groups without counting.

The following activities are appropriate for children who need instruction and who need practice. Give children experiences until they can recognize a variety of groups up to 5 or 6 without counting.

TEACHER-DIRECTED ACTIVITIES	
1:1-13	<i>Tell Me Fast</i>
1:1-10	<i>Hunt for It</i>
1:1-9	<i>Hide it</i>
1:1-12	<i>Find a Match</i>
2:3-2	<i>Instant Recognition of Number Arrangements</i>
2:3-3	<i>Instant Recognition of Number Shapes</i>
2:3-4	<i>Instant Recognition of Number Trains</i>

Ready to Apply (A)

(A) – Recognizes groups without counting.

Go on to focus on learning combination for numbers as described below.

Identifies Parts of Numbers

When children are asked to determine how many altogether in a group of dots, those who add the parts to get the total will identify those parts when they explain how they got the total. For example, “I saw 3 and 2, and that was 5.” However, some children count all the dots. We can’t tell through their explanation whether they can see parts of the number or not. So, we ask, “What groups do you see?”

Needs Prerequisite (N)

(N) – Does not identify groups of 3 or larger.

These children should work with the activities described above for Recognizing Small Numbers.

Needs Instruction (I)

(I) – Is able to see groups of 3 or larger on 2 of the 6 cards.

Provide experiences where children are asked to look for and describe the parts they see.

TEACHER-DIRECTED ACTIVITIES	
2: 3:2	<i>Instant Recognition of Number Arrangements</i>
2: 3-3	<i>Instant Recognition of Number Shapes</i>

INDEPENDENT ACTIVITIES	
2:2-14	<i>Number Arrangements: Using Cubes, Level 1</i>
2:2-15	<i>Number Arrangements: Using Color Tiles, Level 1</i>
2:2-16	<i>Number Arrangements: Using Toothpicks, Level 1</i>
2:2-17	<i>Number Arrangements: Using Collections, Level 1</i>
2:2-18	<i>Counting Boards: Making Up Number-Combination Stories, Level 1</i>

Needs Practice (P)

(P) – Is able to identify groups of 3 on 5 or 6 of the 6 cards.

Focus on finding the largest group they can. Also look for specific groups, including groups of 4 and 5. For example: Show a group of 4 and say, “Can you find a group that looks like this?”

TEACHER-DIRECTED ACTIVITIES	
2:3-2	<i>Instant Recognition of Number Arrangements</i>
2: 3-3	<i>Instant Recognition of Number Shapes</i>

INDEPENDENT ACTIVITIES	
2:2-14	<i>Number Arrangements: Using Cubes, Level 1</i>
2:2-15	<i>Number Arrangements: Using Color Tiles, Level 1</i>
2:2-16	<i>Number Arrangements: Using Toothpicks, Level 1</i>
2:2-17	<i>Number Arrangements: Using Collections, Level 1</i>
2:2-18	<i>Counting Boards: Making Up Number Combination Stories, Level 1</i>

Ready to Apply (A)

(A) – Is able to see parts of 4 or more on at least 2 cards and 3 or more on the rest of the cards.

Combines Parts of Numbers

Needs Prerequisite (N)

(N) – Counts all to determine the total number of dots most of the time (5 or 6 out of 6 cards) or makes 3 or more errors.

Focus on recognizing groups of 2, 3 and 4 dots. Then add in some groups of 5. Present a variety of arrangements made with many different materials.

TEACHER-DIRECTED ACTIVITIES	
2:3-2	<i>Instant Recognition of Number Arrangements (to 5)</i>
2:3-3	<i>Instant Recognition of Number Shapes (to 6)</i>

INDEPENDENT ACTIVITIES	
2:2-14	<i>Number Arrangements: Using Cubes (to 5)</i>
2:2-15	<i>Number Arrangements: Using Color Tiles (to 5)</i>
2:2-16	<i>Number Arrangements: Using Toothpicks (to 5)</i>
2:2-17	<i>Number Arrangements: Using Collections (to 5)</i>

Needs Instruction (I)

(I) – Does not use parts to find the total number of dots for most of the cards; counts all to find out for 4 out of the 6 cards.

Focus on finding the parts they know within the larger numbers.

TEACHER-DIRECTED ACTIVITIES	
2:3-2	<i>Instant Recognition of Number Arrangements</i>
2:3-3	<i>Instant Recognition of Number Shapes</i>

INDEPENDENT ACTIVITIES	
2:2-14	<i>Number Arrangements: Using Cubes, Level 1</i>
2:2-15	<i>Number Arrangements: Using Color Tiles, Level 1</i>
2:2-16	<i>Number Arrangements: Using Toothpicks, Level 1</i>
2:2-17	<i>Number Arrangements: Using Collections, Level 1</i>
2:2-18	<i>Counting Boards: Making Up Number-Combination Stories, Level 1</i>

Needs Practice (P-, P, P+)

At this stage, children are using parts of numbers to find out how many for some of the cards with various levels of proficiency.

(P-) – Begins to recognize they know some groups without counting but still counts all of the dots on 3 out of the 6 cards. May make 2 errors.

(P) – Recognizes more and more groups and only counts all of the dots on 1 or 2 out of the 6 cards. Adds without counting or counts on for the rest of the cards. May make 1 error.

(P+) – Uses parts of numbers to find out how many for all 6 cards. Counts on for 3 cards. May add without counting for more than 3 cards, but makes an error. Does not count all for any of the cards.

Focus on developing an awareness of the combinations they know. Also have them predict how many they think the total will be before counting on.

TEACHER-DIRECTED ACTIVITIES	
2:3-2	<i>Instant Recognition of Number Arrangements</i>
2:3-3	<i>Instant Recognition of Number Shapes</i>

INDEPENDENT ACTIVITIES	
2:2-14	<i>Number Arrangements: Using Cubes, Levels 1 and 2</i>
2:2-15	<i>Number Arrangements: Using Color Tiles, Levels 1 and 2</i>
2:2-16	<i>Number Arrangements: Using Toothpicks, Levels 1 and 2</i>
2:2-17	<i>Number Arrangements: Using Collections, Levels 1 and 2</i>
2:2-18	<i>Counting Boards: Making Up Number-Combination Stories, Level 1</i>
2:2-19	<i>Number Shape Arrangements, Levels 1 and 2</i>
2:2-22	<i>Number-Train Arrangements, Levels 1 and 2</i>
2:3-17	<i>Describing Shape Puzzles</i>
2:2-27	<i>Building and Rebuilding</i>

Ready to Apply

(A) – Uses the parts of numbers to find out how many for all the cards. Adds parts for 4 or more cards. May count on for 2 of the cards but does not count all for any of the cards.

To get more information about what is appropriate for them at the next level, assess using *Assessment 5: Combination Trains*.



5: Combination Trains Assessment

Linking Assessment to instruction

The following charts outline suggested activities for each instructional level of the *Combination Trains* assessment. They refer you to particular activities from the *Developing Number Concepts* (DNC) series of books. The DNC series includes both teacher-directed and independent activities specifically designed to meet the varied needs of students. The suggested activities are coded for easy access. For example, “1:2-23” refers to Book 1, Chapter 2, Activity Number 23.

Please note, these charts provide a quick-overview of activities for instruction. For complete background information, please refer to these other helpful resources:

Math Time: The Learning Environment by Kathy Richardson

Developing Number Concepts by Kathy Richardson

For Professional Development Opportunities

Contact Math Perspectives Teacher Development Center: www.mathperspectives.com

Suggested Reading

For additional information that will support your instruction, read the following sections from *Developing Number Concepts Book Two: Addition and Subtraction*:

- Internalizing Number Combinations to 10 (p. 42-44)
- Goals for Children’s Learning (p. 44)
- Meeting the Range of Needs (p. 47)
- Classroom Scenes (p. 48-53)
- About the Activities (p. 54-55)

Providing Appropriate Instruction

When helping children develop an awareness that number combinations are related and that they can use what they know to figure out what they don’t know, it is important to recognize that competency develops over time. Present a variety of activities, allowing children to experience the concepts in many ways over several weeks. This will help them make generalizations and integrate their ideas.

- **Teacher-Directed Small-Group Work**

Choose 3 or 4 activities and present them in a 10-15 minute small-group session to provide the children with a variety of experiences. Do the same tasks for several days. Occasionally replace one of the tasks

with a new one. Adjust the size of the numbers according to the children’s responses. Usually work with just one group a day so you have time to interact with the children while they work at the Independent Stations.

- **Independent Station Work**

Give children opportunities to choose from among several activities. These choices should be available for several weeks. During this time, the children will be developing proficiency and moving to larger numbers, as they are able. Interact with the children as they work, supporting them and challenging them as needed.

COMBINATION TRAINS

The number combinations in this assessment are presented in a way that makes it possible to find out if children are using what they know about one combination to figure out another. The trains are arranged to assess several levels at the same time. For example, the “thirteen train” is arranged so that the child is asked to add $3 + 3$, $3 + 4$, and $6 + 7$. The results will be organized into Numbers to 6, Numbers to 10 and Numbers to 20, and the instructional level will be determined for each range of numbers.

Needs Prerequisite (N)

(N) – Adds by counting all for all 3 combinations within the range assessed or makes 2 or 3 errors.

To get more information about the children’s ability to identify parts of numbers, use *Assessment 4: Number Arrangements*.

Focus on recognizing small groups of objects. Begin with numbers that are small enough for them to see that they can recognize the groups without counting. Move on to activities listed under “Needs Instruction” when they can identify small groups without counting.

TEACHER DIRECTED ACTIVITIES		To 6	To 10	To 20
1:1-13	<i>Tell Me Fast</i>	•	•	
1:1-10	<i>Hunt for It</i>	•	•	
1:1-9	<i>Hide it</i>	•	•	
1:1-12	<i>Find a Match</i>	•	•	
2:3-2	<i>Instant Recognition of Number Arrangements</i>	•	•	•
2:3-3	<i>Instant Recognition of Number Shapes</i>	•	•	•
2:3-4	<i>Instant Recognition of Number Trains</i>	•	•	•
2:3-3 and 2:3-33	<i>Instant Recognition of Number Shapes Combined with Number-Shape Pairs</i>			•

Needs Instruction (I)

(I) – Adds by counting all for 2 of the 3 combinations within the range being assessed. May have 1 error.

Focus on describing the parts of numbers to help the children move from counting all to combining parts.

TEACHER-DIRECTED ACTIVITIES		To 6	To 10	To 20
2: 2-1	<i>Snap It, Level 1 and Extension</i>	•	•	
2: 2-2	<i>The Tub Game, Level 1 and Extension</i>	•	•	
2: 2-3	<i>The Wall Game</i>	•	•	
2: 2-4	<i>Bulldozer</i>	•	•	
2: 2-5	<i>The Cave Game</i>	•	•	•
2: 2-9	<i>Number Shapes: On and Off</i>	•	•	
2: 2-11	<i>Number Trains: On and Off</i>	•	•	
2: 2-7	<i>Finger Combinations</i>	•	•	
2: 3-1	<i>Combining Stacks: Pick It Up</i>	•	•	•
2: 3-4	<i>Instant Recognition of Number Trains</i>	•	•	
2: 3-3	<i>Instant Recognition of Number Shapes</i>	•	•	
2: 3-3 and 2:3-33	<i>Instant Recognition of Number Shapes Combined with Number-Shape Pairs</i>	•	•	•

INDEPENDENT ACTIVITIES				
2: 2-18	<i>Counting Boards: Making Up Number-Combination Stories</i>	•	•	
2: 3-24	<i>The Tub-Game Station</i>	•	•	
2: 3-25	<i>The Snap-It Station</i>	•	•	
2: 3-18	<i>What Numbers Can You Make?</i>	•	•	•
2: 3-17	<i>Describing Shape Puzzles</i>	•	•	•

Needs Practice (P)

(P) – Adds by counting on for 2 or more of the 3 combinations within the range being assessed. May count all once and may have 1 error.

(P+) – Knows total without counting or uses what is known about one combination to figure out another combination for 2 of the 3 combinations within the range being assessed. May have no errors.

Focus on developing an awareness of the combinations they know. Have them predict how many they think the total will be before counting. Begin with number combinations that are small enough for them to see that they can tell how many altogether without counting.

TEACHER-DIRECTED ACTIVITIES				
		To 6	To 10	To 20
2:2-9	<i>Number Shapes: On and Off</i>	•	•	
2:2-11	<i>Number Trains: On and Off</i>	•	•	
2:3-1	<i>Combining Stacks: Pick It Up</i>	•	•	•
2:3-5	<i>Related Combinations: Short Stacks</i>	•	•	•
2:3-28	<i>Related Combinations: Tall Stacks</i>	•	•	•
2:3-29	<i>How Do You See It? Adding Number Shapes</i>			•
2:3-4	<i>Instant Recognition of Number Trains</i>	•	•	
2:3-3	<i>Instant Recognition of Number Shapes</i>	•	•	
2:3-3 and 2:3-33	<i>Instant Recognition of Number Shapes Combined with Number-Shape Pairs</i>			•

INDEPENDENT ACTIVITIES				
		To 6	To 10	To 20
2:2-20	<i>Number Shapes: Using Number Cubes</i>	•	•	
2:2-21	<i>Number Shapes: Using Spinners</i>	•	•	
2:2-23	<i>Number Trains: Using Number Cubes</i>	•	•	
2:2-24	<i>Number Trains: Using Spinners</i>	•	•	
2:2-25	<i>How Many Ways?</i>	•	•	
2:2-26	<i>Number-Train Graph</i>	•	•	
2:3-14	<i>Combination Toss</i>	•	•	
2:3-17	<i>Describing Shape Puzzles</i>	•	•	•
2:3-18	<i>What Numbers Can You Make?</i>	•	•	•
2:3-21	<i>Grab-Bag Addition Station</i>	•	•	•
2:3-36	<i>Roll and Double</i>			

Ready to Apply (A)

(A) – Knows total without counting or uses what is known about one combination to figure out all 3 of the combinations within the range being assessed. May have no errors.

A child may be “Ready to Apply” for numbers to 6, but not for numbers to 10 and 20. Go on to the next level until the children are “Ready to Apply” for Numbers to 20.



6: Hiding Assessment

Linking Assessment to instruction

The following charts outline suggested activities for each instructional level of the *Hiding Assessment*. They refer you to particular activities from the *Developing Number Concepts* (DNC) series of books. The DNC series includes both teacher-directed and independent activities specifically designed to meet the varied needs of students. The suggested activities are coded for easy access. For example, “1:2-23” refers to Book 1, Chapter 2, Activity Number 23.

Please note – these charts provide a quick-overview of activities for instruction. For complete background information, please refer to these other helpful resources:

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Contact Math Perspectives Teacher Development Center: www.mathperspectives.com

Suggested Reading

For additional information that will support your instruction, read the following sections from *Developing Number Concepts Book Two: Addition and Subtraction*:

- What You Need to Know About Addition and Subtraction (pp. xvii – xxii)
- Interpreting and Symbolizing Addition and Subtraction (pp. 2 – 12)
- Internalizing Number Combinations to 10 (pp. 42 – 53)
- About the Activities (pp. 54-55)
- Developing Strategies for Adding and Subtracting (p. 100 – 111)

Providing Appropriate Instruction

When helping children develop proficiency with decomposing numbers, it is important to recognize that competency develops over time. Present a variety of activities, allowing children to experience working with parts of numbers in many ways over several weeks. This will help them make generalizations and integrate their ideas. Let their responses dictate the amount of instructional time you provide before moving on.

- **Teacher-Directed Small-Group Work**

Choose 3 or 4 activities and present them in a 10-15 minute small-group session to provide the children with a variety of experiences. Do the same tasks for several days. Occasionally replace one of the tasks with a new one. Adjust the size of the numbers according to the children’s responses. Usually work with just one group a day so you have time to interact with the children while they work at the Independent Stations.

- **Independent Station Work**

Give children opportunities to choose from among several activities. These choices should be available for several weeks. During this time, the children will be developing proficiency and moving to larger numbers, as they are able. Interact with the children as they work, supporting them and challenging them as needed.

Part One: Hiding Assessment with Models

Since the goal is for children to know the parts of numbers, the focus for determining the level of instruction they need is on what they know about parts of numbers and not their ability to get right answers. They are “ready to apply” if they know all the parts for the number being assessed. They “need practice” if they know some parts, and count on or use relationships for parts they don’t know. They “need instruction” if they make errors or still count all most of the time. They “need a prerequisite” if they make errors or guess most of the time.

Needs Prerequisite (N)

(N) – Makes three or more errors or guesses.

Provide experiences where children can learn to recognize small groups such as those listed in the following from Developing Number Concepts: Book 2.

Recognizing Small Groups		
TEACHER-DIRECTED ACTIVITIES		Numbers To 5-6
2: 3-2	<i>Instant Recognition of Number Arrangements</i>	•
2: 3-3	<i>Instant Recognition of Number Shapes</i>	•
2: 3-4	<i>Instant Recognition of Number Trains</i>	•

Needs Instruction (I)

(I) – May have 2 errors, counts all for more than half of the combinations.

Provide experiences focused on describing parts of numbers.

TEACHER-DIRECTED ACTIVITIES		To 6	To 10
2:2-1	<i>Snap It, Level 1 and Extension</i>	•	•
2:2-2	<i>The Tub Game, Level 1 and Extension</i>	•	•
2:2-3	<i>The Wall Game</i>	•	•
2:2-4	<i>Bulldozer</i>	•	•
2:2-5	<i>The Cave Game</i>	•	•
2:2-7	<i>Finger Combinations</i>	•	•
2:2-8	<i>Working with Number Shapes</i>	•	•
2:2-9	<i>Number Shapes: On and Off</i>	•	•
2:2-10	<i>Working with Number Trains</i>	•	•
2:2-11	<i>Number Trains: On and Off</i>	•	•
2:2-12	<i>Counting Boards: Number-Combination Stories</i>	•	•

INDEPENDENT ACTIVITIES			
2:2-14	<i>Number Arrangements: Using Cubes</i>	•	•
2:2-15	<i>Number Arrangements: Using Color Tiles</i>	•	•
2:2-16	<i>Number Arrangements: Using Toothpicks</i>	•	•
2:2-17	<i>Number Arrangements: Using Collections</i>	•	•
2:2-18	<i>Counting Boards: Making Up Number-Combinations Stories</i>	•	•
2:2-20	<i>Number Shapes: Using Number Cubes</i>	•	•
2:2-21	<i>Number Shapes: Using Spinners</i>	•	•

Needs Practice (P–, P, P+)

(P–) – Counts all for up to half of the combinations, may have 1 error.

(P) – Figures out 2 or more, may have 1 error, may not count all.

(P+) – Knows all but 1 quickly, no errors, no counting all (may count on or back or use relationships for 1 combination).

Focus on identifying missing parts of numbers using objects.

TEACHER-DIRECTED ACTIVITIES		To 6	To 10
2:2-1	<i>Snap It, Level 2</i>	•	•
2:2-2	<i>The Tub Game, Level 2</i>	•	•
2:2-5	<i>The Cave Game</i>	•	•
2:2-6	<i>Grab-Bag Subtraction</i>	•	•

INDEPENDENT ACTIVITIES			
2:3-15	<i>Build-a-Floor Race</i>	•	•
2:3-16	<i>Apartment Buildings</i>		•
2:3-22	<i>Grab-Bag Subtraction Station</i>	•	•
2:3-25	<i>The Snap-It Station</i>	•	•
2:3-26	<i>What's Missing?</i>	•	•

Ready to Apply (A)

(A) – Knows all quickly, no errors.

Begin work with identifying missing parts mentally as described in Part Two: Hiding Assessment without Models.

Part Two: Hiding Assessment without Models

The expectation is that the child will be “Ready to Apply” on Part One for any of the numbers you will be assessing in Part Two.

Even when children know parts of numbers with a model, many find the task much harder when there is no model to use as a referent. It is a different level of challenge for young children to think about objects without anything to refer to. When a model is hidden, it is still there and adds support for the children’s thinking. One part is visible which means they don’t have to visualize that part. “What if” questions require the child to imagine both parts.

Needs Prerequisite (N)

(N) – Makes three or more errors or guesses.

Needs Instruction (I)

(I) – May have 2 errors, counts all for more than half of the combinations.

Children who need a prerequisite or need instruction, most likely need more experiences with the model. Continue to provide experiences with the model even for those numbers they knew (ready to apply) on Part One.

Needs Practice (P-, P, P+)

(P-) – Counts all for up to half of the combinations, may have 1 error.

(P) – Figures out 2 or more, may have 1 error, may not count all.

(P+) – Knows all but 1 quickly, no errors, no counting all (may count on or back or use relationships for 1 combination).

Provide practice identifying the missing parts mentally. Focus on numbers they know very well first. Increase the numbers as they are able.

TEACHER-DIRECTED ACTIVITIES		To 6	To 10
2:3-6	<i>What Do You Think? Using Counting Boards</i>	•	•
2:3-7	<i>What Do You Think? Using Grab Bags</i>	•	•
2:3-8	<i>What Do You Think? Using Tubs</i>	•	•
2:3-9	<i>Let's Pretend: Grab Bags</i>	•	•
2:3-10	<i>Let's Pretend: Counting Boards</i>	•	•
2:3-11	<i>Let's Pretend: Number Trains</i>	•	•
2:3-12	<i>Let's Pretend: Number Shapes</i>	•	•

INDEPENDENT ACTIVITIES			
2:3-20	<i>Counting Boards: Think and Write</i>	•	•

Ready to Apply (A)

(A) – Knows all quickly, no errors.

If children are ready to apply their knowledge of the parts of numbers:

Provide opportunities to use what they know to solve problems using two-digit numbers.



7: Ten Frames Assessment

Linking Assessment to instruction

The following charts outline suggested activities for each instructional level of the *Ten Frames* assessment. They refer you to particular activities from the *Developing Number Concepts* (DNC) series of books. The DNC series includes both teacher-directed and independent activities specifically designed to meet the varied needs of students. The suggested activities are coded for easy access. For example, “1:2-23” refers to Book 1, Chapter 2, Activity Number 23.

Please note, these charts provide a quick-overview of activities for instruction. For complete background information, please refer to these other helpful resources:

Math Time: The Learning Environment by Kathy Richardson

Developing Number Concepts by Kathy Richardson

For Professional Development Opportunities

Contact Math Perspectives Teacher Development Center: www.mathperspectives.com

Suggested Reading

For additional information that will support your instruction, read the following sections from

From *Developing Number Concepts Book Two: Addition and Subtraction*:

- Internalizing Number Combinations to 10 (pp. 42-44, 54-55)
- About Number Combinations to 20 (p.151)

Providing Appropriate Instruction

When you are helping students develop proficiency adding and subtracting by making or breaking up a ten, it is important to recognize that competency develops over time. To be proficient, students must know parts of numbers to 10, understand teen numbers as a ten and some ones, and be able to mentally reorganize two single-digit numbers into a ten and leftovers and to know immediately the number they end up with.

Provide opportunities over several weeks for students to work with the idea of forming tens and leftover ones and breaking up tens in many ways. At the same time, provide additional experiences with number combinations, if needed. Let student’s responses dictate the amount of instructional time you spend before moving on.

- **Teacher-Directed Small-Group Work**

Choose 3 or 4 activities and present them in a 10-15 minute small-group session to provide the children with a variety of experiences. Do the same tasks for several days. Occasionally replace one of the tasks with a new one. Adjust the size of the numbers according to the children's responses. Usually work with just one group a day so you have time to interact with the children while they work at the Independent Stations.

- **Independent Station Work**

Give children opportunities to choose from among several activities. These choices should be available for several weeks. During this time, the children will be developing proficiency and moving to larger numbers, as they are able. Interact with the children as they work, supporting them and challenging them as needed.

TEN FRAMES

Children must have several skills in place in order to fluently add and subtract by forming and breaking up tens. They must be able to think of numbers from 11-19 as one ten and some more. In order to use the structure of tens and ones efficiently, they also need to know the parts of numbers through 10 and to be able to compose and decompose teen numbers. Whether the children are asked to add or subtract, they must be able to hold several steps in mind as they complete the process. Competency with all these skills develops over time. It is important to provide whatever time is necessary, because these skills are foundational to children's understanding of numbers to 99 and multi-digit addition and subtraction.

Ten Frames: Addition

In this assessment, children are asked to combine two numbers. While there are many different strategies children could bring to these tasks, this assessment poses questions deliberately worded to find out what children know about making tens and leftovers and to check their ability to use what they have learned about number relationships and combinations up to ten when solving these problems. For addition, the children have a model (the ten frame) of the number they start with, but do not have a model of the number being added. This requires them to mentally decompose the number they are adding to determine how many are needed to make a ten and how many are left.

1. Addition: Adding Ones to a Ten ($10 + 9$, $6 + 10$)

Needs Prerequisite (N)

(N) – Is unable to add to 10 so counts all the stars, or makes 2 errors.

*In order to get more information about what the children understand about numbers, assess using **Assessment 5: Number Combinations** and **Assessment 4: Number Arrangements**, and work with the tasks recommended.*

Needs Instruction (I) and Needs Practice (P)

Needs Instruction (I)

(I) – Knows either $10 + 9$ or $6 + 10$ and counts all to figure out the other one, **or** makes an error.

Needs Practice (P)

(P) – Knows either $10 + 9$ or $6 + 10$, but is able to count on from ten for the one not known.

The children who need instruction or need practice have a limited understanding of numbers as tens and ones. Focus on teacher-directed experiences asking them to combine one ten with various leftovers. This will help them begin to recognize the pattern that emerges when ten is added to a single digit number.

Teacher Directed Activities	
2:3-30	<i>Working with Ten-Shapes Addition: Ten Plus a Number</i>
2:3-31	<i>A Ten-Shape and More: Subtraction (Do the readiness activity described with this activity which focuses on addition.)</i>

Ready To Apply (A)

(A) – Knows both $10 + 9$ and $6 + 10$ without any counting or errors. Has the skills necessary to go on to combining numbers by making a ten and leftovers.

2. Knows Parts of Numbers

When children add by making a ten and leftovers, they need to know the parts of ten and the parts of the numbers being added to ten.

Needs Prerequisite (N)

(N) – Does not know the parts of numbers so counts all or guesses or makes 2 or more errors.

*To get more information about what the child knows about parts of numbers use **Assessments 4: Number Arrangements** and **Assessment 5: Combination Trains**.*

Needs Instruction (I)

(I) – Knows 1 of the 4 parts of numbers.

*To get more information about what the children know, assess using **Assessment 6: Hiding**.*

Focus on describing the parts of numbers

TEACHER-DIRECTED ACTIVITIES			
		To 6	To 10
2:2-1	<i>Snap It</i>	•	•
2:2-2	<i>The Tub Game</i>	•	•
2:2-5	<i>The Cave Game</i>	•	•
2:2-6	<i>Grab-Bag Subtraction</i>	•	•
2:2-8	<i>Working with Number Shapes</i>	•	•
2:2-9	<i>Number Shapes: On and Off</i>	•	•
2:2-21	<i>Number Shapes: Using Spinners</i>	•	•

INDEPENDENT ACTIVITIES			
		To 6	To 10
2:3-15	<i>Build-a-Floor Race</i>	•	•
2:3-19	<i>Addition-and-Subtraction Spin It</i>	•	•
2:3-20	<i>Counting Boards: Think and Write</i>	•	•
2:3-21	<i>Grab-Bag Addition Station</i>	•	•
2:3-22	<i>Grab-Bag Subtraction Station</i>	•	•
2:3-26	<i>What's Missing?</i>	•	•

Needs Practice (P)

(P-)– Know parts for 2 of the items with 1 error.

(P) – Knows 2 with no errors or knows 3 or 4 with 1 error.

(P+)– Knows 3 and counts on for 1 with no errors.

Children who need practice know some of the parts of numbers. Focus on asking the children to predict ahead of time what they think the missing part will be.

TEACHER-DIRECTED ACTIVITIES			
		To 6	To 10
2:2-1	<i>Snap It, Level 2</i>	•	•
2:2-2	<i>The Tub Game, Level 2</i>	•	•
2:2-5	<i>The Cave Game</i>	•	•
2:2-6	<i>Grab-Bag Subtraction Station</i>	•	•

INDEPENDENT ACTIVITIES			
		To 6	To 10
2:3-15	<i>Build-a-Floor Race</i>	•	•
2:3-16	<i>Apartment Buildings</i>		•
2:3-22	<i>Grab-Bag Subtraction Station</i>	•	•
2:3-25	<i>The Snap-It Station</i>	•	•
2:3-26	<i>What's Missing?</i>	•	•

Ready to Apply (A)

(A) – Knows all 4 of the parts necessary to solve the problems and uses them with ease to determine the number of tens and leftovers.

3. Making a Ten and Adding Ones

Children can often add 10 and a number of ones with ease, but find it much more difficult if they have had to break up numbers to make the ten with some leftovers. They can't just rely on the pattern of adding 10 and some more but must hold the ten in their mind as a unit.

The children need to work with the following activities listed that focus them on making tens and combining them with ones no matter what their instructional level. They will need differing amounts of support to do the tasks and differing amounts of time working with these ideas depending on their instructional level.

Needs Prerequisite (N)

(N) – Counts all or counts on from the number of stars in the ten frame for 2 (out of 3).

Children at this level show little or no understanding of tens and ones. They are still thinking of numbers as a collection of ones instead of as groups of tens and leftover ones, so they count all or count on from the number of stars in the ten frame for 2 out of the 3 items.

Needs Instruction (I)

(I) – Adds by counting on from 10 for all three items, or by counting on from the number of stars or counting all the stars for 1 or 2 of the items.

Children at this level have a very beginning understanding of combining a ten and some ones.

Needs Practice (P-, P, P+)

(P-) – Combines tens and ones without counting for 2 items but makes an error or counts all the stars for one item.

(P) – Either counts on from 10 for 2 of the items or counts from the number of stars for one item.

(P+) – Knows 2 parts and counts on from 10 to figure out 1 of the items.

Children at this level understand how to combine a ten and some ones, but it is not yet easy, or is not always accurate.

First, focus on breaking up the numbers to make a ten; "How many do we need to make a ten?"

Then, focus on how many would be left; "If you used this part to make a ten, how many would be left?"

After, focus on what the total would be; "What do you notice about the part that is left and the total?"

TEACHER-DIRECTED ACTIVITIES	
2:3-30	<i>Working with Ten-Shapes</i> <i>Focus on Addition</i>
2:3-31	<i>A Ten-Shape and More: Subtraction</i> <i>(Do the readiness activity described with this activity which focuses on addition.)</i>
2:3-33	<i>Number Shape Pairs (Adapt–have children combine by making a ten and leftovers).</i> <i>Break the number to be added on (8 in this example) into the parts necessary to make a ten.</i> <i>Example: 7 + 8</i> $\begin{array}{c} \wedge \\ 3 \quad 5 \end{array}$ <i>Add the 3 to make a ten: $7 + 3 = 10$</i> <i>Add the part left over (5 in this example) to find the total: $10 + 5 = 15$</i> <i>(Make sure they understand before having them work with this as an independent task.)</i>

INDEPENDENT ACTIVITIES	
2:3-33	<i>Number Shape Pairs (Adapt–have children write how many to make a ten as described above in Teacher-Directed Activities.)</i> <i>Example: 7 + 5</i> $7 + 3 = 10$ $\begin{array}{c} \wedge \\ 3 \quad 2 \end{array}$ $10 + 2 = 12$
2:3-34	<i>Two Ten-Shapes: Addition Station (Focus on Addition)</i>

Ready to Apply (A)

(A) – Knows the totals without counting for all of the items.

Ten Frames: Subtraction

1. Subtracting a Ten From Ones (15 is 10 and 5, 14 is 10 and 4, 13 is 10 and 3)

Needs Prerequisite (N)

(N) – Knows 0 (out of 3), or has 2 or more errors.

Children at this level do not have an understanding of decomposing teen numbers into a ten and some ones. Make sure they can combine tens and ones before working with them on decomposition. Then, explore what happens to numbers when ten is taken away.

TEACHER-DIRECTED ACTIVITIES	
2:3-30	<i>Working with Ten-Shapes – Subtraction: Minus Ten</i>
2:3-31	<i>A Ten-Shape and More: Subtraction</i>

Needs Instruction (I) and Needs Practice (P)

Children at this level have a beginning understanding of the composition of teen numbers as 1 ten and some ones.

Needs Instruction (I)

(I) – Knows 1 (out of 3), may have 1 error.

Needs Practice (P)

(P-)– Knows 2 (out of 3) and counts on 1 (out of 3), with 1 error.

(P) – Knows 2 (out of 3) and counts on 1 (out of 3), no errors.

For all levels, focus on decomposing numbers from 11 to 19 into 1 ten and the ones that are left over. The following activities use ten frames. Sometimes use connecting cubes to organize cubes into one ten and some ones in addition to using ten frames

TEACHER-DIRECTED ACTIVITIES	
2:3-30	<i>Working with Ten-Shapes: Subtraction</i>
2:3-31	<i>A Ten-Shape and More: Subtraction</i>

INDEPENDENT ACTIVITIES	
2:3-34	<i>Two Ten-Shapes: Addition and Subtraction</i>
2:3-35	<i>A Ten-Shape and More: Subtraction Station</i>
2:3-37	<i>Wipe Out</i>

Ready to Apply

(A) – Knows 3 (out of 3), no errors.

Knows Parts of Numbers

When children subtract by breaking up a number in order to get to ten and then take the rest of the number away from the ten, they need to know the parts of ten and the parts of the numbers being taken from ten. Children may appear to know the parts to 10 when assessed on the *Hiding Assessment*, but in *Ten Frames*, they have to apply what they know about parts.

The Instructional Levels and the Tasks are the same as for Ten Frames: Addition

Making a Ten and Subtracting Ones

For subtraction, students decompose a teen number into one ten and leftovers. They subtract by breaking up a number to get to ten and take the remaining part from the ten.

Needs Prerequisite (N)

(N) – Knows 0, or has 2 or more errors.

Needs Instruction (I)

(I) – Knows 1 (out of 3), may have 1 error.

Needs Practice (P)

(P) – Knows 3 (out of 3) with 1 error, **or** knows 2 (out of 3) and counts 1.

(P-)– Knows 2 (out of 3), with 1 error.

First, focus on breaking up the teen number to make a ten, “How many do we need to take away to get to ten?” Example: “We have 14. We are going to take 6 away. How many do we need to take away to get to 10?” ($14 - 4 = 10$)

Then, focus on the part that would be left, “We have 14. We are going to take away 6. We take away 4 to get ten. How many do we still need to take away? 6 is 4 and 2. We have to take 2 away.”

After, focus on what the remainder would be; “How many of the ten will be left if we take 2 away?”

TEACHER-DIRECTED ACTIVITIES	
2:3-30	<i>Working with Ten Shapes: Subtraction</i>
2:3-31	<i>A Ten-Shape and More: Subtraction</i>

INDEPENDENT ACTIVITIES	
2:3-34	<i>Two Ten-Shapes: Addition and Subtraction</i>
2:3-35	<i>A Ten-Shape and More: Subtraction Station</i>
2:3-37	<i>Wipe Out</i>

Ready to Apply (A)

(A) – Knows 3 (of 3), no errors.

3. Extension: Ten More (Addition and Subtraction)

This question is designed to see whether children recognize the relationship between "8 + 7" and "18 + 7," and "13 - 6" and "23 - 6." The same experiences can be used for both addition and subtraction. All students who reach this level of the assessment need to work with more than 1 ten frame to see what happens with the numbers.

Needs Prerequisite (N)

(N) – Is unable to do the task or needs to count all to determine how many.

Needs Instruction (I)

(I) – Counts on/back to determine how many.

Needs Practice (P)

(P) – Makes twenty with 3 left over when adding, **or** subtracts 3 from 23.

Ready to Apply

(A) – Adds ten to the previous total.

Work with the following tasks but adapt them by adding one or more ten frames. Focus on what happens to the numbers when ten frames are added.

Examples: $12 - 6 = 6$ $22 - 6 = ?$ $32 - 6 = ?$

TEACHER-DIRECTED ACTIVITIES	
2:3-30	<i>Working with Ten-Shapes (Adapt by adding another ten frame)</i> <i>Addition: Ten Plus a Number</i> <i>Subtraction: Minus Ten</i>
2:3-31	<i>A Ten-Shape and More: Subtraction (Adapt by adding one or more additional ten frames.)</i>

INDEPENDENT ACTIVITIES	
2:3-34	<i>Two Ten-Shapes: Addition Station (Adapt by adding one or more additional ten frames.)</i>
2:3-35	<i>A Ten-Shape and More: Addition Station (Adapt by adding one or more additional ten frames.)</i>



8: Grouping Tens Assessment

Linking Assessment to instruction

The following charts outline suggested activities for each of the instructional levels for the *Grouping Tens* assessment. They refer you to particular activities from the *Developing Number Concepts* (DNC) series of books. The DNC series includes both teacher-directed and independent activities specifically designed to meet the varying needs of students. The suggested activities in each DNC book are coded for easy access. For example, “1:2-23” refers to Book 1, Chapter 2, Activity Number 23.

Please note that these charts provide a quick-overview of activities for instruction. For complete background information, please refer to these other helpful resources:

Math Time: The Learning Environment by Kathy Richardson

Developing Number Concepts by Kathy Richardson

For Professional Development Opportunities

Contact Math Perspectives Teacher Development Center: www.mathperspectives.com

Suggested Reading

For additional information that will support your instruction, read the following sections from *Developing Number Concepts Book Three: Place Value, Multiplication and Division*:

- What You Need to Know About Place Value (p. 2-4)
- Goals for Children’s Learning (p.7-8)
- Meeting the Range of Needs (p. 9)
- Classroom Scenes (p. 10-13)
- About the Activities (p. 14)

Providing Appropriate Instruction

When helping children develop proficiency with numbers composed of tens and ones, it is important to recognize that competency develops over time. Present a variety of activities, allowing children to experience organizing numbers into tens and ones in many different situations over several weeks. This will help them make generalizations and integrate their ideas about the underlying structure of numbers. Let their responses dictate the amount of time you spend before moving on.

- **Teacher-Directed Small Group Work**

Choose 3 or 4 activities and present them in a 10-15 minute small-group session to provide the children with a variety of experiences. Do the same tasks for several days. Occasionally replace one of the tasks

with a new one. Adjust the size of the numbers according to the children’s responses. Usually work with just one group a day so you have time to interact with the children while they work at the Independent Stations.

- **Independent Station Work**

Give children opportunities to choose from among several activities. These choices should be available for several weeks. During this time, the children will be developing proficiency and moving to larger numbers, as they are able. Interact with the children as they work, supporting them and challenging them as needed.

Level 1: Tens and Ones to 20

This section of the assessment determines whether students can decompose numbers from 11 to 19 into 1 ten and some leftover ones, and whether they understand that the 1 in the tens places represents ten objects.

There are 3 responses that determine the instructional level:

1. Number of tens in 17
2. Number of leftover ones in 17
3. Number of objects the 1 in 13 represents

Needs Prerequisite (N)

(N) Knows 0 (out of 3) responses.

Children at this level are unable to answer any of the assessment questions correctly. Focus on teacher-directed experiences asking them to combine one ten with various leftovers. This will help them begin to recognize the pattern that emerges when ten is added to a single-digit number.

*To get more information about the child’s understanding of teen numbers, assess using **Assessment 7: Ten Frames.***

TEACHER DIRECTED ACTIVITIES	
2:3-30	<i>Working with the Ten-Shapes</i>
	<i>Addition: Ten Plus a Number</i>
	<i>Subtraction: Minus Ten</i>
2: 3-31	<i>A Ten-Shape and More: Subtraction (Do the readiness activity described within this activity.)</i>

Needs Instruction (I)

(I) Knows 1 (out of 3) responses

Children who need instruction do not fully understand that teen numbers are made up of one ten and some ones and do not understand what is represented by the symbols. Focus on decomposing numbers from 11 to

19 into 1 ten and the ones that are left over and building quantities to represent the symbols. The following activities use ten frames. Sometimes use connecting cubes to organize cubes into one ten and some ones in addition to ten frames.

TEACHER-DIRECTED ACTIVITIES	
2:3-30	<i>Working with the Ten-Shapes: Addition</i>
2:3-31	<i>A Ten-Shape and More: Subtraction</i>

INDEPENDENT ACTIVITIES	
2:3-34	<i>Two Ten-Shapes: Addition and Subtraction</i>
2:3-35	<i>A Ten-Shape and More: Subtraction Station</i>

Needs Practice (P)

(P) Knows 2 (out of 3) responses

Children are at this stage if they have some understanding of the structure of teen numbers but need more experiences to clarify and strengthen this concept. Focus on predicting the answers before actually building the numbers with the counters. Sometimes use connecting cubes instead of ten frames.

TEACHER-DIRECTED ACTIVITIES	
2:3-30	<i>Working with the Ten-Shapes: Addition and Subtraction</i>
	<i>Addition: Nine Plus a Number</i>
	<i>Addition: Eight Plus a Number</i>
	<i>Addition: Mixing Them Up</i>
	<i>Subtraction: Minus Nine</i>
2:3-31	<i>A Ten-Shape and More: Subtraction</i>

INDEPENDENT ACTIVITIES	
2:3-34	<i>Two Ten-Shapes: Addition and Subtraction</i>
2:3-35	<i>A Ten-Shape and More: Subtraction Station</i>

Ready to Apply (A)

(A) Knows 3 (out of 3), no errors.

Children who are "Ready to Apply" understand the number of tens and ones in the "teen" numbers and should move on to work with the structure of numbers as tens and ones to 100 as described in the following section.

Level 2: Tens and Ones to 100

This section of the assessment determines whether the students can tell how many altogether when they know how many tens and ones there are and if they can add ten and subtract ten without counting.

Many of the same activities are used for all the levels, but the way children work with these activities will vary depending on their level of understanding and the interactions the teacher has with them while they are working.

Needs Prerequisite (N)

(N) Counts all to combine tens and ones.

Children are at this level if they do not use the concept of tens and ones to find out how many but instead count all of the counters. Continue to work with Tens and Ones to 20 as long as needed. Also give them experiences learning to count groups and look for patterns. Focus on forming and counting groups.

TEACHER-DIRECTED ACTIVITIES	
3:1-1 to 1-10	<i>The Process of Regrouping (Grouping Games), Discovering Number Patterns, and Pattern in Base Ten, p. 15-34</i>
2:3-31	<i>A Ten-Shape and More: Subtraction</i>

INDEPENDENT ACTIVITIES	
3: 1-17	<i>Recording Various Number Patterns on Strips</i>
3:1-18	<i>Grab and Add</i>

Needs Instruction (I)

(I) Counts by tens to combine tens and ones.

Children who need instruction have some awareness of tens as they are able to count by tens to get to the total number of counters. However, they are not thinking of ten as a unit since they count on to add and take away 10 from 34. Give them experiences learning to count groups and look for patterns. Also give them a variety of experiences organizing tens and ones in many different situations until they see the relationship between the particular number of tens and ones and the total number of objects.

TEACHER-DIRECTED ACTIVITIES	
3:1-1 to 1-10	<i>The Process of Regrouping (Grouping Games), Discovering Number Patterns, and Pattern in Base Ten, p. 15-34</i>

INDEPENDENT ACTIVITIES	
3:1-32	<i>Lots of Lines, Level 1</i>
3:1-33	<i>Paper Shapes, Level 1</i>
3:1-34	<i>Yarn, Level 1</i>
3:1-35	<i>Yarn Shapes</i>
3:1-36	<i>Containers, Level 1</i>
3:1-37	<i>Cover It Up Level 1</i>
3:1-38	<i>Measuring Things in the Room, Level 1</i>
3:1-39	<i>Measuring Myself, Level 1</i>

(continued on next page)

3:1-40	<i>Comparing Myself</i>
3:1-41	<i>Making Trails</i>
3:1-42	<i>Building Stacks</i>
3:1-43	<i>Race to 100</i>
3:1-44	<i>Race to Zero</i>

Needs Practice (P-, P, P+)

Children are at this level when they can combine tens and ones to get a total without counting. However, they do not fully understand that ten is a unit since they need to count to add or take away one ten.

(P-) Combines 3 tens and 4 ones without counting but needs to count to add 10, **and** take 10 away.

(P) Combines tens and ones without counting, but needs to count to either add 10, **or** take 10 away.

(P+) Does not correctly tell the number of tens in the estimate; knows all other responses.

Provide experiences with the Grouping Games if they have not already experienced these. Also give them a variety of experiences organizing tens and ones in many different situations until they can add or take away ten as a unit without needing to count. As they work, occasionally ask them how many there would be if they added 10 or took 10 away.

TEACHER-DIRECTED ACTIVITIES	
3:1-1 to 1-10	<i>The Process of Regrouping (Grouping Games), Discovering Number Patterns, and Pattern in Base Ten, p. 15-34 (If not previously experienced.)</i>

INDEPENDENT ACTIVITIES	
3:1-32	<i>Lots of Lines, Level 1</i>
3:1-33	<i>Paper Shapes, Level 1</i>
3:1-34	<i>Yarn, Level 1</i>
3:1-35	<i>Yarn Shapes</i>
3:1-36	<i>Containers, Level 1</i>
3:1-37	<i>Cover It Up Level 1</i>
3:1-38	<i>Measuring Things in the Room, Level 1</i>
3:1-39	<i>Measuring Myself, Level 1</i>
3:1-40	<i>Comparing Myself</i>
3:1-41	<i>Making Trails</i>
3:1-42	<i>Building Stacks</i>
3:1-43	<i>Race to 100</i>
3:1-44	<i>Race to Zero</i>

Children are at this level when they can combine tens and ones and add and take away without counting. However, they are not able to tell the number of tens in a 2-digit number. Give them experiences where they start with a symbolic number and build it with connecting cubes and cups of beans.

Ready to Apply (A)

(A) Knows numbers of tens in estimate; Combines tens and ones without counting; Adds 10 without counting; subtracts 10 without counting; no errors.

Children who are “Ready to Apply” understand the structure of numbers as tens and ones and should move on to work adding and subtracting groups of tens as described in the following section.

Level 3: Adds/Subtracts Groups of Tens

This section of the assessment determines whether the students can add and subtract groups of tens without counting. If they are able to think of tens as units, they will be able to add 3 tens as easily as 3 ones and take away 4 tens as easily as they take 4 ones away. If children are not “Ready to Apply” adding or subtracting groups of tens, they need to continue working with the activities described for Tens and Ones to 100 with some variations.

Needs Prerequisite (N)

(N) Counts by 1s to add and/or subtract groups of tens OR makes 2 errors when adding and subtracting groups of tens.

At this stage, the children are not able to add groups of tens. Have the children continue to work with Numbers to 20 or Numbers to 100 at the appropriate level according to the assessment.

Needs Instruction (I)

(I) Counts by 10s to add and subtract groups of tens, **or** makes 1 error when adding or subtracting groups of tens.

Needs Practice (P)

(P) Correctly adds groups of tens, **or** correctly subtracts groups of tens without counting.

At this stage, the child who “Needs Instruction (I)” counts by 10s to add and subtract. The child who “Needs Practice (P)” counts by tens to either add or to subtract. These children have not yet fully recognized that adding 10s is like adding 1s if you consider the tens a unit, or they need practice to develop facility.

As they work, occasionally ask them how many there would be if they added groups of tens or took groups of tens away. After they have finished finding out the number of tens and ones in the task they are doing, have them add groups of tens and/or take groups of tens away.

INDEPENDENT ACTIVITIES	
Vary the tasks by having the children add or subtract groups of tens when working with the activities.	
3:1-32	<i>Lots of Lines, Level 1 Vary by adding groups of tens</i>
3:1-33	<i>Paper Shapes, Level 1 Vary by adding groups of tens</i>
3:1-34	<i>Yarn, Level 1 Vary by adding groups of tens</i>
3:1-35	<i>Yarn Shapes Vary by adding groups of tens</i>
3:1-36	<i>Containers, Level 1 Vary by adding groups of tens</i>
3:1-37	<i>Cover It Up Level 1 Vary by adding groups of tens</i>

Ready to Apply (A)

(A) Knows numbers of tens in estimate; combines tens and ones without counting; adds 10 without counting; subtracts 10 without counting; no errors.

These children are ready to begin adding and comparing groups as an extension of what they have been working on before. You can provide a challenge for these children by having them work at Levels 2 and 3 which ask them to compare.

INDEPENDENT ACTIVITIES	
3:1-32	<i>Lots of Lines, Levels 2 and 3</i>
3:1-33	<i>Paper Shapes, Levels 2 and 3</i>
3:1-34	<i>Yarn, Levels 2 and 3</i>
3:1-35	<i>Yarn Shapes, Extension</i>
3:1-36	<i>Containers, Levels 2 and 3</i>
3:3-37	<i>Cover It Up, Levels 2 and 3</i>
3:1-38	<i>Measuring Things in the Room, Levels 2 and 3</i>
3:1-39	<i>Measuring Myself, Levels 2 and 3</i>
3:1-40	<i>Comparing Myself</i>



9: Two Digit Addition and Subtraction

Linking Assessment to instruction

The following charts outline suggested activities for each the instructional levels for the *Two-Digit Addition and Subtraction* assessment. They refer you to particular activities from the *Developing Number Concepts* (DNC) series of books. The DNC series includes both teacher-directed and independent activities specifically designed to meet the varying needs of students. The suggested activities in each DNC book are coded for easy access. For example, “1:2-23” refers to Book 1, Chapter 2, Activity Number 23.

Please note that these charts provide a quick-overview of activities for instruction. For complete background information, please refer to these other important resources:

Math Time: The Learning Environment by Kathy Richardson

Developing Number Concepts by Kathy Richardson

For Professional Development Opportunities

Contact Math Perspectives Teacher Development Center: www.mathperspectives.com

Suggested Reading

For additional information that will support your instruction, refer to these sections from *Developing Number Concepts Book Three: Place Value, Multiplication, and Division*.

- What You Need to Know About Place Value (p. 2–4)
- Developing a Sense of Quantities to 100 and Beyond (p. 57–59)
- Addition and Subtraction of Two-Digit Numbers (p. 99–103)
- A Classroom Scene (p. 104–107)

Providing Appropriate Instruction

When helping students develop proficiency with two-digit addition and subtraction, it is important to recognize that competency develops over time and is the integration of several mathematical skills. To be proficient, the student must understand that numbers are composed of tens and ones; they must know parts of numbers to 10; and they must be able to reorganize quantities of tens and ones. In addition, they need to move away from using models and understand what is happening when they combine and separate numbers to solve problems without models.

- **Teacher-Directed Small Group Work**

Choose 3 or 4 activities and present them in a 10-15 minute small-group session to provide the children with a variety of experiences. Do the same tasks for several days. Occasionally replace one of the tasks with a new one. Adjust the size of the numbers according to the children's responses. Usually work with just one group a day so you have time to interact with the children while they work at the Independent Stations.

- **Independent Station Work**

Give children opportunities to choose from among several activities. These choices should be available for several weeks. During this time, the children will be developing proficiency and moving to larger numbers, as they are able. Interact with the children as they work, supporting them and challenging them as needed.

TWO-DIGIT ADDITION AND SUBTRACTION

This assessment helps you analyze precisely what students know and need to learn when working with two-digit addition and subtraction. The assessment provides information about how students solve the problems and whether or not they need a model to do so.

Goal:

To determine if the student can use the concept of tens and ones to add and subtract two-digit numbers by mentally breaking them apart and reorganizing them into tens and ones.

Addition:

The problems are presented at 3 different levels of abstraction:

Solving problems with a model ($28 + 16$, $26 + 27$)

Solving problem without a model ($36 + 25$)

Solving a symbolic problem ($28 + 14$)

Subtraction:

The problems are presented at 3 different levels of abstraction:

Solving problems with a model ($33 - 14$, $53 - 27$)

Solving problem without a model ($41 - 26$)

Solving a symbolic problem ($34 - 16$)

Present a variety of activities, allowing students to experience two-digit addition and subtraction in many ways over several months while providing additional experiences, if needed, with place value and number combinations. Let their responses dictate the amount of instructional time you spend before moving on.

Two-Digit Addition and Subtraction

Begin focusing primarily on addition.

Many of the same activities are used at all levels, but the focus changes according to the needs of the students. The following experiences focus on both addition and subtraction. Work primarily on addition first, but give students experiences with subtraction so they can begin to see how they are related. Once students are proficient with addition, focus primarily on subtraction, but continue to provide some addition practice as well.

1: Solving Problems with a Model

The assessment first determines whether students can solve problems when they have a model to refer to but are asked not to touch or move the model.

Needs Prerequisite (N)

(N) – Touches or moves model for both problems, counts all, or makes 2 errors

At this level, the children are unable to solve the problems using tens and ones.

*To get additional information about what the student understands about tens and ones and to determine what instruction the student needs in order to be ready to work with two-digit addition, use **Assessment 7: Ten Frames: Addition** and **Assessment 8: Grouping Tens**. If students are having difficulty breaking numbers to 10 apart to make tens, use **Assessment 6: Hiding Assessment** to find out what they know about parts of numbers.*

The following activities help students develop an understanding that numbers are composed of tens and ones. Students should work with these experiences until they can combine tens and ones and can add 10 and take 10 away without counting.

The Grouping Games

3:1-1	<i>Introducing the Plus-One and Minus-One Games</i>
3:1-2	<i>The Grouping Games with Groups of Other Sizes</i>
3:1-3	<i>Plus or Minus Any Number</i>
3:1-4	<i>Regrouping Beyond Two Places</i>

INDEPENDENT ACTIVITIES	
3:1-32	<i>Lots of Lines, Level 1</i>
3:1-33	<i>Paper Shapes, Level 1</i>
3:1-34	<i>Yarn, Level 1</i>
3: 1-35	<i>Yarn Shapes</i>
3:1-36	<i>Containers, Level 1</i>
3:1-37	<i>Cover It Up, Level 1</i>

(continued on next page)

3:1-38	<i>Measuring Things in the Room, Level 1</i>
3:1-39	<i>Measuring Myself, Level 1</i>
3:1-40	<i>Comparing Myself</i>
3:1-41	<i>Making Trails</i>
3:1-42	<i>Building Stacks</i>

Needs Instruction (I)

(I) – Counts to solve 1 problem, or touches/moves the model 1 time, or solves 1 problem as though it was written down and is incorrect for 1 problem.

Students need instruction if they show some beginning but tentative understanding of tens and ones. Focus on addition of two-digit numbers using models so the students can see what happens when the numbers are combined or broken apart and recombined.

TEACHER-DIRECTED ACTIVITIES	
2:3-30	<i>Working with Ten-Shapes</i>
3:1-45	<i>Addition and Subtraction of Two-Digit Numbers</i>
3:1-46	<i>Story Problems</i>

The students should also work to strengthen their understanding of Numbers as Tens and Ones through the following learning experiences. The students should work with these experiences until they can combine tens and ones and can add 10 and take 10 away without counting.

INDEPENDENT ACTIVITIES	
3:1-32	<i>Lots of Lines, Level 1</i>
3:1-33	<i>Paper Shapes, Level 1</i>
3:1-34	<i>Yarn, Level 1</i>
3: 1-35	<i>Yarn Shapes</i>
3:1-36	<i>Containers, Level 1</i>
3:1-37	<i>Cover It Up, Level 1</i>
3:1-38	<i>Measuring Things in the Room, Level 1</i>
3:1-39	<i>Measuring Myself, Level 1</i>
3:1-40	<i>Comparing Myself</i>
3:1-41	<i>Making Trails</i>
3:1-42	<i>Building Stacks</i>

Needs Practice (P-, P, P+)

Students need practice if they have to figure out the combinations needed to make tens and leftover ones and do not always easily keep in mind the number of tens they have formed or they ignore the models and solve the problems as though they were written down.

(P-) – Ignores the tens and ones represented by the model and visualizes the problem as though it were written down for 1 or 2 problems, or makes an error.

Children who ignore the tens and ones represented by the models and visualize the problem as though it were written down for 1 or 2 problems need more practice to see how tens and ones work. They also need more practice if they make an error when solving the problems using tens and ones.

(P) – Children are at this level if they can reorganize the numbers into tens and ones but count to figure out the parts for both problems, no errors.

(P+) – Children are at this level if they use tens and ones to solve the problems but need to count to figure out the parts for one of the problems.

To get more information if students are having difficulty breaking numbers to 10 apart to make tens, use **Assessment 6: Hiding Assessment** to find out what they know about parts of numbers and **Assessment 7: Ten Frames** to see if they can use parts of numbers to make one ten and leftovers.

Use the following activities to give the students practice breaking numbers apart and reorganizing them into tens and ones. Focus on helping them think about what number they need to make a ten and the number of leftovers they will have. Ask, “What do you think it will be?” and then have the students check if necessary. If they count to figure out, use the cubes and focus on adding parts of numbers rather than adding on the number by counting one by one.

TEACHER-DIRECTED ACTIVITIES	
2:3-30	<i>Working with Ten-Shapes</i>
3:1-45	<i>Addition and Subtraction of Two-Digit Numbers</i>
3:1-46	<i>Story Problems</i>

INDEPENDENT ACTIVITIES	
2:3-34	<i>Two Ten-Shapes: Addition Station</i>
3:1-48	<i>Partner Add-It</i>
3:1-50	<i>Roll and Add</i>
3:1-52	<i>Add 'Em Up: Lots of Lines</i>
3:1-53	<i>Add 'Em Up: Paper Shapes</i>
3:1-54	<i>Add 'Em Up: Measuring Things in the Room</i>
3:1-55	<i>Add 'Em Up: Yarn</i>
3:1-56	<i>Add 'Em Up: Yarn Shapes</i>
3:1-57	<i>Add 'Em Up: Containers</i>
3:1-58	<i>Add 'Em Up: Cover It Up</i>
3:1-59	<i>Solving Story Problems</i>

Ready to Apply (A)

Students are “Ready to Apply (A)” when they know the number combinations needed to make tens and leftovers and arrive at the answer without having to count to figure anything out and without errors. Move on to 2. Solving a Problem without a Model activities if the student is not yet “Ready to Apply” at that level.

2: Solving a Problem without a Model (36 + 25)

The students who are able to solve the first two problems in *Assessment 9: Two-Digit Addition & Subtraction* without touching or moving the model are asked to solve a problem when the model is hidden.

Needs Prerequisite (N)

(N) – Students need a prerequisite if they remove the paper to look at the models and then touch or move the model in order to see how to reorganize them into tens and ones.

*They need to continue to use the activities listed in the previous section **using models** until they can solve problems by looking at but not touching the model. Provide practice using the same activities as listed above with the emphasis now on moving away from dependence on the model.*

Needs Instruction (I)

(I) – Students are at this level if they remove the paper and look at the model in order to imagine what will happen to the numbers when they reorganize them into tens and ones.

*Work with the students with the activities listed above until they can easily solve the problems without touching the models. Then, begin asking them to try and solve parts of the problem **without looking at the models and then check** using the models. It will be helpful to some students if you **begin by adding single-digit numbers or work with tens only until it is easy for them to add these numbers without a model.***

Needs Practice (P-, P)

Students need practice if they can solve the problem without the model but are not yet proficient reorganizing the tens and ones to get to the answer.

(P-) – Students at this level ignore the tens and ones and try to solve the problem by visualizing it as though it were written on paper.

They should work with the models until they understand what is happening and can solve problems by mentally organizing numbers into tens and ones.

(P) – Students at this level can solve the problem but need to figure out the parts of the numbers in order to do so.

Use the same activities listed above. Focus on asking the children to think about what is needed to make a ten and the number of leftovers they will have. Have them solve parts of the problem without looking at the models. If necessary, have them check using the models.

TEACHER-DIRECTED ACTIVITIES	
2:3-30	<i>Working with Ten-Shapes</i>
3:1-45	<i>Addition and Subtraction of Two-Digit Numbers</i>
3:1-46	<i>Story Problems</i>

INDEPENDENT ACTIVITIES	
2:3-34	<i>Two Ten-Shapes: Addition Station</i>
3:1-48	<i>Partner Add-It</i>
3:1-50	<i>Roll and Add</i>
3:1-52	<i>Add 'Em: Up Lots of Lines</i>
3:1-53	<i>Add 'Em Up: Paper Shapes</i>
3:1-54	<i>Add 'Em Up: Measuring Things in the Room</i>
3:1-55	<i>Add 'Em Up: Yarn</i>
3:1-56	<i>Add 'Em Up: Yarn Shapes</i>
3:1-57	<i>Add 'Em Up: Containers</i>
3:1-58	<i>Add 'Em Up: Cover It Up</i>
3:1-59	<i>Solving Story Problems</i>

Ready to Apply (A)

(A) – Students have reached the desired level of proficiency when they solve problems without the need for a model, without having to count to figure anything out, and without errors.

Move on to 3. Solving Symbolic activities if the student is not yet “Ready to Apply” at that level.

TEACHER-DIRECTED ACTIVITIES	
2:3-30	<i>Working with Ten-Shapes</i>
3:1-45	<i>Addition and Subtraction of Two-Digit Numbers</i>
3:1-46	<i>Story Problems</i>

3: Solving Symbolic Problems

This part of the assessment focuses on the student’s ability to solve symbolic problems correctly and to demonstrate an understanding of the process used by showing the process with a model.

Needs Prerequisite (N) and Needs Instruction (I)

(N) – Gets the answer wrong and is unable to show the process they used.

(I) – Gets the correct answer but can’t show the process used.

Needs Practice (P)

The following activities are many of the same used previously, but emphasize interpreting symbolic problems with models.

(P-) – Is able to solve the problem and shows what they did with the model, but makes an error.

(P) – Is able to reorganize the numbers into tens and ones but counts to figure out the parts for both problems.

The goal is to work with the activities to develop proficiency with the process of using tens and ones to solve symbolic problems. Ask the students to describe how many are needed to make a ten and leftovers.

If students count to determine the number needed, have them use models and describe the parts they use to make the ten and what parts are leftover. When they can describe the parts with ease when they are using a model, ask them to imagine the parts they need by just looking at the written symbols. Continue to alternate between using the models and not using the models depending on how proficient the students are.

TEACHER-DIRECTED ACTIVITIES	
3:1-45	Addition and Subtraction of Two-Digit Numbers
3:1-47	Figure it Out

INDEPENDENT ACTIVITIES	
Have the children record, not only the answer, but how they got the answer as well.	
3:1-48	Partner Add-It
3:1-50	Roll and Add

Ready to Apply (A)

(A) – Is able to use tens and ones without counting or using a standard algorithm and shows with a model how the process used works.

If the student is “Ready to Apply” for addition problems, move on to an emphasis on subtraction. If the students are able to solve a variety of problems requiring two-digit addition or subtraction, they can begin working with forming and counting hundreds.

Additional Notes on Two-Digit Subtraction

Needs Prerequisite or Needs Instruction

It is not typical for a child to need a prerequisite for subtraction if they are doing well with addition using tens and ones. Help them make the connection to what they know about tens and ones.

Have those students who are struggling with subtraction and are not ready to work with subtraction independently, continue to work with the addition stations described in the previous section, Two-Digit Addition.

Needs Practice

The students should work with subtraction of two-digit numbers using models to see what happens when the numbers are combined or broken apart and recombined. Make sure the student understands they are taking away from the whole number and do not need to build the number they are subtracting. Some children will have used counting backwards to subtract and may be reluctant to give that method up. Begin by taking groups of ten away such as $34 - 10$ or $52 - 30$ to help them see there are more efficient and easier ways to solve problems than counting back. Taking groups of tens away will also help the students realize that it can be easier to take away from a number if they do not break it into tens and ones to subtract.

When asking children to solve a problem like $43 - 24$, the children will often break 43 into 40 and 3 to take the tens away and will describe what they did by saying, “I took 2 tens from 40, and now I have 2 tens left.” Focus them on the fact they still have some ones by asking, “How many did you start with? Tell me the whole number, not just the tens.” “So when you take 2 tens from 43, how many do you have left?”

Including the ones in the answer may not be easy for some children who are focused on taking tens from tens. They may need to be reminded for quite a while before they automatically include the ones in their answer.

Children take away the remaining ones in different ways (in this problem, they need to take away 4.) Some will break up a ten and then combine what is left with the 3 ones.

“I took the 4 from the ten and that left 6. Then I put the 6 with the 3 ones and that made 9. I have 1 ten and 9. That makes 19.”

Some will take away as many of the ones as they can and then take the rest from the tens.

“I can take these 3 ones away, but I still have to take away one more. I take it from the ten and now I have 19 left.”

Children generally have more difficulty solving subtraction problems than addition problems, so continue to have them use models to see what happens to the numbers until that is easy. Then ask them to say what they think will happen before they move the models and have them check and see. Eventually, they will be able to subtract easily by just looking at the models without touching or moving them. At this point, you can move on to covering the models. Let the children uncover the model if they have tried to solve the problem and realize they need to look. Alternate between showing the models, covering the models, and not using the models, depending on how proficient the students are.

The following activities focus on subtraction.

INDEPENDENT ACTIVITIES	
2:3-34	<i>Two Ten-Shapes: Subtraction</i>
2:3-35	<i>A Ten-Shape and More: Subtraction Station</i>
3:1-49	<i>Partner Take-Away</i>
3:1-51	<i>Roll and Subtract</i>
3:1-59	<i>Solving Story Problems</i>

When children are proficient subtracting by “taking away”, they can begin working with subtraction as the difference between two quantities. Use the following activities.

3: 1-32	<i>Lots of Lines, Level 2</i>
3: 1-33	<i>Paper Shapes, Level 2</i>
3: 1-34	<i>Yarn, Level 2</i>
3:1-36	<i>Containers, Level 2</i>